European University Institute

DEPARTMENT OF ECONOMICS

Three Essays on Reputation, Household Debt, and Monetary Policy

Clodomiro F. Ferreira Mayorga

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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- I confirm that chapter 1 was jointly co-authored with **Ms. Emanuela Iancu** (National Institute of Economic Research, Sweden), and I contributed with 50% of the work.
- I confirm that chapter 2 was jointly co-authored with **Mr. James Cloyne** (Bank of England) and **Paolo Surico** (London Business School), and that I contributed with 50% of the work.

Signed and date:

Clodomiro F. Ferreira Mayorga
April 26th, 2016

Abstract

This dissertation is composed of three separate and self-contained chapters on two different areas: (i) reputation building and competition in on-line markets, and (ii) the heterogeneous transmission of monetary policy to household consumption expenditures and income.

The first paper investigates how sellers' strategic competition for high valuation buyers shapes reputation building incentives in a setting resembling an on-line market, and how it determines the dynamics of prices and reputation itself. Sellers repeatedly auction off a good to a pool of short-lived buyers; efforts and valuations are private information. Ceteris paribus, as the reputation of competitors increase (intensity of competition), a seller's incentive to exert effort that helps to successfully complete a transaction decrease. This "intensity" of competition effect, however, quickly disappears as the number of buyers in the market increases, providing a motivation for the use of equilibrium concepts such as "oblivious equilibrium", in which the only payoff relevant reputation is the average one.

The second chapter shifts focus to household expenditure, debt and monetary policy. It is shown that, in response to an interest rate change, mortgagors in the U.K. and U.S. adjust their spending significantly (especially on durable goods) but outright home-owners do not. While the dollar change in mortgage payments is nearly three times larger in the U.K. than in the U.S., these magnitudes are much smaller than the overall change in expenditure. In contrast, the income change is sizable and similar across both household groups and countries. Consistent with the predictions of a simple heterogeneous agents model with credit-constrained households and multi-period fixed-rate debt contracts, our evidence suggests that the general equilibrium effect of monetary policy on income is quantitatively more important than the direct effect on cash-flows.

The third chapter exploits individual mortgage data in the UK to try to further understand the role of mortgagor's balance sheets in the transmission of monetary policy. Estimation results point in the direction of significant heterogeneity in two dimensions: (i) in the response of observed leverage (loan-to-value) and affordability (loan-to-income) ratios at the time of origination for the median mortgagor, and (ii) the response of LTVs for households that are first-time-buyers and those that are non-first-time-buyers.

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The value and uniqueness of a journey is usually to be found in the traveling itself as well as in the people whom you travel with, not in the point of destination. By this token, I can say with no room for doubt that my Ph.D. years at the EUI, as well as at the Bank of England and London Business School, have constituted an unforgettable journey. All in all, the dissertation that emerged from it would not have been possible without the vital academic and personal support of many people.

First, I wish to express my gratitude to my supervisors, Russell Cooper and Piero Gottardi, for their patience, suggestions and support at different stages of the dissertation. Both taught me, among other things, that research can be tough and rewarding at the same time. In addition, Arpad Abraham and Fernando Vega-Redondo provided insightful comments and support along the way. Of course, both chapter 1 and chapter 2 would not have existed without my co-authors Ema Iancu, James Cloyne and Paolo Surico. I have learned a great deal and had a great time working side-by-side with them.

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Finally, I want to dedicate this thesis to Flori. No words are enough to express what you have ment to me all these years.

Contents

Chapter 1

Reputation and Strategic Competition: The Case of an Online Market

joint with **Emanuela Iancu** (National Institute of Economic Research, Sweden)[1]

Keywords: reputation, strategic competition, on-line market, private information

1.1 Introduction

Many markets, especially on-line markets such as eBay, Amazon or on-line house renting platforms, are characterized by (at least) two key informational features. First, there is a potential role for reputation building: buyers have little knowledge about the sellers or the quality of a product they are interested in purchasing. Feedbacks and comments by previous buyers, as well as the information content in sellers' marketing campaign, can then be aggregated into public *reputation measures*. In the spirit of Akerlof, 1970, such measures might help welfare improving trades to occur, and in the process allow "good" sellers to increase their revenues. Second, precisely this public and non-anonymous nature of reputation can give rise to strategic behavior that affects observed market outcomes.

This paper tackles two relevant questions related to the incentives characteristic for on-line markets. We first investigate the role of *strategic competition* between sellers in altering the incentives to *build a reputation*, and how this affects observed market outcomes. To our knowledge, this is the first paper to characterize the interaction between strategic competition and reputation building. Second, we analyze quantitatively two identification problems in the reduced-form estimation of the impact of reputation on a seller's revenues. These problems can arise when strategic or reputation building behavior is not incorporated into the estimated model, even when panel data or field experiment data on revenues and reputation measures is used.

[1]This chapter has benefited greatly from comments and discussions with Arpad Abraham, Jean-Pierre Benoît, Wouter Den Haan, Daniel García, David Levine, David Pothier, Tomás Rodriguez Barraquer, David Strauss, Fernando Vega-Redondo, Felipe Zurita, as well as seminar participants at the World Congress of Game Theory 2012, the EEA-ESEM Meetings in Malaga 2012, the EUI Netwroks and Microeconomics Working Groups, EUI Cooper Meetings.

In our benchmark environment, two long lived sellers repeatedly auction homogeneous items to a finite number of different bidders each period. Over time, sellers compete in attracting a higher number of high valuation bidders. Sellers have different abilities, a feature that is only privately observed. In addition, once the good is sold, sellers put effort in delivering the item (i.e. send the promised good on time, without quality deficiencies, etc.), a decision unobserved by other buyers and sellers. The game therefore presents both adverse selection and moral hazard problems. Buyers have private valuations for the good on auction, and do not have any incentive to purchase more than one object over time. Once the transaction is completed, the winner of the auction *truthfully* reports the outcome in a binary form, an information which is added to the public history of the seller. Buyers form beliefs in a Bayesian fashion about sellers' unobserved abilities and anticipate the optimal effort decision for each type of seller. Based on these measures, they optimally select an auction and place a bid as described in Bapna, Dellarocas, and Rice, 2010.

Our first finding points out the fact that when sellers compete strategically, competition erodes marginal future profits and hampers sellers incentive to exert high effort in the first place: the higher the current reputation of the competitor, the less effort is a seller willing to put in order to build a reputation for the future. Nevertheless, we find that, as the number of bidders in the market increases, the effect of competition on effort exertion is reduced. In addition, a high number of potential buyers every period incentivizes sellers to work more in the present as they expect higher future profits (i.e. a higher highest expected bid in their auction). Our results on reputation building per se fall in line with Cripps, Mailath, and Samuelson, 2004 and Holmstrom, 1999 predictions. Independent on whether a retailer acts as a single seller or faces competition from another fellow, a seller will always choose a relative higher effort to improve on his reputation when the latter is low. Once his type has been learned by the market, he opts for a relatively lower effort. This has been coined as the "spending of reputation".

Our second contribution consists in analyzing two identification problems that are linked to the reduced form estimation of the impact of reputation on revenues when strategic interactions and reputation building behavior are not taken into account in the estimation process. In addition, we analyze the dependence of such bias on market characteristics, such as the number of bidders at any given point.

Recent empirical studies have estimated different reduced form effects of reputation and feedback changes on sellers' revenues / sales in on-line markets, using both randomized "field" experiment data and observational panel data on revenues and reputation. Resnick et al., 2006 presents the first field experiment study in which a seller with an established reputation is asked to create a second identity and to operate (i.e. sell) under both identities employing the same "effort". The researchers then compare different revenue measures generated under the two identities.[2] This strategy allows them to control for many potential factors omitted in observational studies, related specially to the effort in presenting the good, in answering inquiries, in packaging, etc. They show that, indeed, the identity with better reputation generates higher revenues: willingness to pay increases by 8%, although they also find that one or two negative feedback does ot have a significant effect on revenues for the "new" seller. In observational data study, Cabral and Hortaçsu, 2010 analyze a sample of 819 sellers of three different goods[3] and follow them at monthly intervals between October 24, 2002 and March 16, 2003. They first estimate a reduced form impact of reputation on prices using cross sectional data; in a second exercise, they use longitudinal data of seller's sales and reputation histories to estimate the impact of the *first negative feedback* on the dynamics of sales. They find that conditional mean (age-detrended) sales growth rates decrease from $+5\%$ to -8% immediately after the *first* negative.[4]

We claim, however, that such reduced form (linear) estimations of the impact of reputation miss two crucial aspects related to reputation building and strategic competition: both sellers' reputations and unobserved effort decisions might affect current revenues of a given seller, being at the same time correlated with the seller reputation. This feature is central to our model, as we consider it to have a significant impact on a seller's behavior. In order to understand the importance of such omission, we study the infinite horizon version of the game described above. This allows us to focus on the dynamics of revenues, efforts and reputations. We modify the computational algorithm presented in Doraszelski and Pakes, 2007, and solve numerically for a Markov Perfect Equilibrium. We then use simulated histories of revenues, reputation and effort to characterize the problems in identifying the reputation-elasticity of revenues from not incorporating reputational information and unobserved effort decisions into a reduced form model. In our dynamic game, the relevant elasticity is non-monotone on a seller's reputation, and it varies with the competitor's reputation. Due to the strategic behavior, reputations are correlated in equilibrium, which implies that the omission of the competitors reputation in the estimated model will bias the results. The mechanism that leads to this bias arises if the decisions made by a seller's competitors depend on the seller's reputation, and if buyers somehow understand this.

The paper proceeds in the following way: in section 1.2 we briefly review our contribution from the point of view of recent papers on the topic. In section 1.3 we lay out the two-period environment and we define the equilibrium concept employed throughout the paper. section 1.4 then develops the main analysis, characterizing the

[2]They also document 15 previous observational studies of the impact of reputation on different measures of revenues.

[3] IBM ThinkPad T23, collectible coins and 1998 Holidays Teddy Beanie Babies.

[4]In order to estimate these effects, Cabral and Hortaçsu, 2010 averaged weekly sales growth rates over a four week window before and after the week in which the seller got his first and respectively second, third, fourth and fifth negative feedback. Then they conduct t-tests for equality of means before and after the feedback. They use both a cross-section regression and a panel data approach to estimate the effect of reputation. The results, in line with previous research, show a steady positive trend of profits until the first negative feedback is received.

mechanisms relevant for our results in a two period version of the game.[5] Finally, section 1.5 concludes.

1.2 Related Literature

The present paper makes a significant contribution to a recently emerged strand from the reputation literature, that analyzes competition among long-lived firms in a dynamic environment dominated by adverse selection and moral hazard problems. Although the role of reputation building has been studied extensively since the seminal paper of Kreps and Wilson, 1982, there is only a small number of studies analyzing games with asymmetric information in which many long lived players compete against each other. This literature, fostered by Horner, 2002, Tadelis, 2002, Vial, 2010 and Vial and Zurita, 2012, builds on the one-seller case studied in the papers of Holmstrom, 1999 and Mailath and Samuelson, 2001. The purpose of these models is understanding whether competition can induce a reputational equilibrium in which opportunistic players choose high effort. The way in which these studies model competition, however, is through Walrasian markets. In consequence, the strategic behavior of the long-run players becomes rather limited, which implies that sellers do not incorporate in their decisions the market's beliefs about their competitors. We contribute to this literature by assuming a dynamic auction environment with a finite number of sellers and buyers; this allows us to explicitly analyze the role of *strategic competition* in shaping sellers' reputation building decisions.

Cripps, Mailath, and Samuelson, 2004 and Holmstrom, 1999 show that within a monopolistic environment a reputational mechanism cannot survive indefinitely as a mean to sustain an equilibrium with a high level of effort if the uncertainty about types disappears over time. Reputation - in the sense of Kreps and Wilson, 1982- is a commitment device that allows sellers to overcome the moral hazard problem, characterized by the short-term incentive to exert low effort, even though higher profits could be earned when committing to high effort in the long run. The dynamics of reputation in our benchmark game between a monopolist and a sequence of N-bidders sets is subject to the same pattern: as long as there is uncertainty about one's type, sellers exert costly effort in equilibrium. Over time, the market learns the seller's type and so the incentives to employ costly effort gradually decrease. As in Holmstrom, 1999, the type of a seller has an intrinsic value, in addition to the information that it provides about the effort decision of a seller in equilibrium.

Our benchmark game stems from Mailath and Samuelson, 2001, with some important distinctions. They consider an environment with imperfect public monitoring, in which consumers observe only a noisy signal of firms' decision. "Inept" sellers can

[5] In the appendix to this chapter, section A.5 which is preliminary work, I describe and briefly characterize an infinite horizon version of the game in order to assess quantitatively the the problems to empirically identify the reputation elasticity of revenues.

only exert low effort while the "competent" have the choice between high or low effort. Reputation building then becomes an exercise where competent firms try to separate themselves from inept ones, rather than mimicking a "Stackelberg" type. The key ingredient for maintaining high effort on the equilibrium path is the replenishment of uncertainty by adding an unobservable type change process. With permanent types, they find that there are no pure strategy Markov equilibrium with high effort is exerted in equilibrium. The key idea behind these results lies in the convergence of high and low effort value functions when uncertainty is not replenished or when firms are replaced always by competent ones. Our benchmark game does not present such uncertainty in types, and we can still generate temporarily high effort. The reason lies in that, unlike Holmstrom, 1999 or our paper, in Mailath and Samuelson, 2001 a high reputation is valuable only to the extent that it signals high effort.

Horner, 2002, Vial, 2010 and Vial and Zurita, 2012 analyze reputational equilibrium in competitive environments. In a setup in which both firms and consumers are atomistic and infinitely lived as described in Horner, 2002, competition supports the existence of equilibria in which opportunistic firms always exert high effort. In Horner, 2002 reputation is built gradually and vanishes instantly upon the deliverance of bad quality, for which a firm is punished by losing its customers to other firms with impeccable records. Prices are initially low but rise with a firm's reputation. Vial, 2010 and Vial and Zurita, 2012 consider a competitive environment as in Mailath and Samuelson, 2001 and show that a reputational equilibrium with high effort can be attained. In contrast with Horner, 2002, in Vial, 2010 and Vial and Zurita, 2012 reputation builds and dissipates gradually. Nevertheless, while Vial, 2010 focuses on the population distribution of reputations and its impact on equilibrium prices, Vial and Zurita, 2012 investigates the entry/exit decision of a firm and the resulting industry dynamics. As in Horner, 2002, sellers in our model profit more not only from having *a good* reputation, but from having *the better* reputation. Under the assumption that the number of buyers is sufficiently higher relative to the number of sellers, we ensure that sellers will always expect a certain number of bidders with positive probability. The allocation of bidders to auctions in equilibrium is done in an assortative fashion: bidders with high valuation bid exclusively in the auction of the seller with the *relatively higher* chances to successfully deliver the good, while bidders with valuations from the lower part of the distribution bid with positive probability in both auctions. The threshold that separates the "high valuation" bidders from the "low valuation" bidders is endogenously determined by the likelihood ratio of successful deliveries of the two sellers. This result ensures a gradual increase and respectively a gradual dissipation of reputation as in Vial, 2010. This very aspect will allow us to investigate the strategic role competition plays in our model in contrast with the above mentioned papers.

As in Vial, 2010 and in our model, the equilibrium price function is key to understanding the incentives of sellers to exert effort, both in the monopolistic and in the competitive setting. The equilibrium price is a function of both equilibrium beliefs about a seller's type and the distribution of buyers' valuations and it incorporates the equilibrium behavior of both parties. In this setting, the updating of beliefs after a success or failure determines both the effect of current actions on future reputations and implicitly on future revenues. The main difference with respect to Vial, 2010 and Horner, 2002 is our focus on the *finite* number of bidders and sellers, that induces long-run players to strategically compete for high valuation buyers.

1.3 The Set-Up

The general game builds on Mailath and Samuelson, 2001 and introduces some modifications. In the baseline game, time is discrete, and indexed by $t = 1, 2$. In each period, there are either 1 or 2 long-lived seller(s) that open an auction each, and N new risk-neutral buyers that select (if more than 1 seller) and bid once in the auctions available at the time they arrive to the market. A good is allocated to a buyer through a second-price auction. All auctions start and end simultaneously. Further, the set of buyers will be denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$, while the set of sellers by $\mathcal{J} = \{1, 2\}$.

1.3.1 Sellers

There are 2 risk-neutral long-run sellers that each period put up for sale a homogeneous good. Sellers are long-lived, with a *common* discount factor δ. A seller can be of either of"good" or of "bad" type. The type is indexed by $\theta \in \{g, b\}$, where $0 \leq b \leq g \leq 1$ and stands for the *privately* observed innate ability or trustworthiness of the seller to execute a successful transaction. The *prior* probability that a seller is of good type is $\mu_{j,0} \equiv P\left(\theta_j = g\right)$.

Assumption 1 *The type θ_j is fixed throughout the life of the seller j, for $j = 1, 2$.*

Assumption 1 implies that there is no "replenishment" of uncertainty. This is different from Mailath and Samuelson, 2001, where types are allowed to change randomly or endogneously.

In our setting, we think about the type of a seller as an "aggregate" measure of *competence* and *morality* characteristics.[6]

[6]When interpreting a seller's type, we don't distinguish among two *dimensions* of her privately observed characteristics: her *trustworthiness* in revealing information before trade takes place and her *ability* to deliver the good in a successful way. Jullien and Park, 2011 analyzes the emergence of reputational equilibrium when these two dimensions are distinguished. They propose an equilibrium where there is an endogenous complementarity, in terms of market perception of information, between honesty in pre-trade communication (honesty) and ability to deliver good quality (reputation). In such setting, pre-trade communication is *cheap-talk*; however, in the benchmark equilibrium they consider, high type sellers truthfully reveal the quality of their good, while low types tend to *lie* more often; once the quality of the good is revealed (through a feedback system) a truthfull pre-trade communication is mapped into a higher probability of the seller being of high ability. Therefore, learning occurs at a faster rate than when the distinction in the dimensions of reputation is not consider (our paper).

Actions, Information and Payoffs

For successfully delivering the product after the auction has been carried out, a seller has to exert some effort e. We think about the effort as the amount of work put in delivering the product properly, securely and professionally packaged, in sending in the specified time the exact advertised object. Effort has a cost for the seller, $c(\cdot)$, and is *unobserved* by buyers and by the other seller (i.e. the competitor).

Assumption 2 *The cost function is increasing and convex and has the following properties:* $c(e^\theta) = 0$, $c'(e^\theta) = 0$ $\forall e^\theta \in [0, \underline{e}]$ *and* $c(e^\theta) > 0$, $c'(e^\theta) > 0$ *and* $c''(e^\theta) > 0$ $\forall e^\theta \in (\underline{e}, 1)$. *The cost function is the* same *for all sellers.*

In other words, for a sequence of low efforts, the seller incurs no costs; effort becomes costly above the threshold e̲, which we call the "maximum costless effort". By devising a cost function of this form, we are implicitly assuming that it is a weakly dominant strategy for each seller to exert a positive amount of effort, to which a strictly positive probability of success is associated [7].

Each transaction carried out by seller j and buyer i at time t has an *outcome*, which we define as a random variable $y_{t,j}$ that can take two values: S (success) or F (failure).

Assumption 3 *The technology is such that the probability of success depends both on sellers' effort* and *on his type, i.e. the probability of success of each transaction takes the form*

$$P\left(y_{j,t} = S|e_{t,j}^{\theta_j}, \theta_j\right) = e_{t,j}^{\theta_j} \cdot \theta_j \tag{1.1}$$

For a given effort level, the "good" seller has almost always a higher probability of success in delivering the product. Conditional on effort and types, outcomes are independent across sellers:

$$P\left(Y_j = y_j, Y_{-j} = y_{-j}\middle|\theta_j, \theta_{-j}; e_j^{\theta_j}, e_{-j}^{\theta_{-j}}\right) = P\left(Y_j = y_j\middle|\theta_j, e_j^{\theta_j}\right) P\left(Y_{-j} = y_{-j}\middle|\theta_{-j}, e_{-j}^{\theta_{-j}}\right) \quad \forall j \neq -j \tag{1.2}$$

At the beginning of each period t, sellers have two sets of information: *public* information, given by the publicly observable sequence of outcomes $\{y^{t-1}\}_{t\geq 1}$ with $y^{t-1} = \{y_{1,t-1}, y_{2,t-1}\}$ assuming that $y_{j,0} = \emptyset$ $\forall j \in 1, 2$, and *private* information, given by his type (which is time-invariant) and the sequence of own effort decisions $\left\{e_{j,t}^{\theta_j}, \theta_j\right\}_{j,t\geq 1}$. [8]

The public *reputation* of seller j at the beginning of auction $t \geq 1$ is defined as

$$\mu_{j,t-1} \equiv P\left(\theta_j = g|y^{t-1}\right)$$

[7] Note that this is a similar assumption to a standard one in the literature, which allows for 0-effort but assumes that the probability of success (or more generally, the density of outcomes) is always greater than 0. In particular, many papers assume two possible effort levels, say e^L and e^H, and then assume that the density of outcomes given e^H first-order stochastically dominates the one given e^L.

[8] Here we assume that prices paid by previous buyers in seller's j past auctions are not observed by upcoming buyers. This rules out complicated behavior and equilibria in which buyers condition their bidding on past realized prices.

where $\mu_{j,0}$ is the belief about the seller's j type at the beginning of the game. Given our assumption regarding the public information y^{t-1} about seller j at the beginning of period t, $\mu_{j,t-1}$ is the *common* belief that all other players (buyers and other sellers) hold about seller j being of good type.

Let $\mathcal{H}_j^t$ be the set of all possible public histories seller j can have at the beginning of period $t+1$: $\mathcal{H}_j^t = \{h_j^t \in \{S,F\}^t, t \geq 1\}$, where $h_j^0 = \emptyset$. The set of all possible histories of the game is defined by the product of all sellers' histories $\mathcal{H}_t = \underset{j}{\times}\mathcal{H}_j^t$.

Seller's j expected revenue from an auction, R_j, is the expected second highest bid of the respective auction paid by the winning bidder. In a second price auction, the expected revenue of the seller is given by

$$\boldsymbol{R}_j = \mathbb{E}\left(b_j^{(2)} \middle| \text{\# of buyers, strategies of buyers}\right)$$

Henceforth, the expected revenue of seller j at time t denoted by $R_{j,t}$ is a function of all sellers' equilibrium actions in period t. Under the assumption that each seller sells only one good each period, seller's j ex-ante stage payoffs are given by

$$\pi_{j,t}^{\theta_j} = \boldsymbol{R}_{j,t} - c\left(e_{j,t}^{\theta_j}\right) \tag{1.3}$$

The inter-temporal problem for seller j of type θ_j is then

$$\max_{\left\{e_{j,t}^{\theta_j}\right\}_{t=1,2}} \quad \Pi_{j,0}^{\theta_j} = \boldsymbol{R}_{j,1} - c\left(e_{j,1}^{\theta_j}\right) + \delta\mathbb{E}_1\Big(\boldsymbol{R}_{j,2} - c\left(e_{j,2}^{\theta_j}\right)\Big) \tag{1.4}$$

1.3.2 Buyers

The benchmark setting follows closely Bapna, Dellarocas, and Rice, 2010. Different from them, however, in our setup buyers anticipate seller's equilibrium actions and incorporate this information in their optimal bidding strategies.

Buyers have *unit demand* and only live for one period, i.e. they are interested in buying one unit of the good offered by sellers today (in later sections we will discuss deviations from this assumption). Each buyer has private information about his valuation v^i, and valuations are independently drawn from the distribution $v^i \sim U\,[0,1]$.

Assumption 4 *The total number of bidders N is constant in every period. In addition, both N as well as the distribution from which valuations are drawn every period are* common *knowledge to all market participants.*

Actions and Payoffs

Buyers' strategies consist in deciding in which auction to bid, and the bid $b_j^i \in [0,\infty)$ they place in the respective auction. Therefore, a buyer's i strategy consists of two elements: a *bid vector* $b^i(v^i)$ and an *auction selection strategy* $s^i(v^i)$. The *bid vector* $b^i =$

$(b_1^i, \ldots, b_J^i)$ consists of elements b_j^i representing the amount of money buyer i is willing to place in seller's j auction if she were to bid in that auction. An *auction selection strategy* is a vector $s^i(v^i) = (s_j^i(v^i))_{j=1}^J$ where $s_j^i(v^i)$ is the probability that buyer i with valuation v^i bids in seller's j auction.

Assumption 5 *In our benchmark game, buyers are allowed to select only* ***one auction*** *and to bid once.*

Since buyers' valuations are freshly drawn every period, their strategic behavior is independent of time and hence we omit the time index in the buyers' problem.

Prior to selecting a seller, each buyer forms beliefs about sellers delivering successfully the good. We make a final assumption about the information available to buyers:

Assumption 6 *Current bidders cannot observe past prices.*

Assumption 6 implies that buyers only observe past outcomes before deciding. Given sellers' reputations $\mu = (\mu_j)_{j \in \mathcal{J}}$, buyers anticipate the effort a seller j of type θ_j will exert. The belief regarding the successful delivery of seller j is defined as

$$r_{j,t} \equiv P\left(y_{j,t} = S | \mu_{j,t-1}\right) = \mu_{j,t-1} \cdot g\tilde{e}_{j,t}^g + (1 - \mu_{j,t-1}) \cdot b\tilde{e}_{j,t}^b \tag{1.5}$$

where $\tilde{e}_{j,t}^{\theta_j}$ is the anticipated effort of seller j in auction t if she were of type θ_j. In other words, $r_{j,t}$ stands for the anticipated probability of a successful transaction when facing a seller of unknown type whose public beliefs are by $\mu_{j,t-1}$.

It is a well established result that in a second price auction truthful bidding is a weakly dominant strategy. In our setup buyers' optimal strategy is to bid their expected valuation of the item in each auction:

$$b_j^{*i}(v^i, r_{j,t}) = r_{j,t} v^i \tag{1.6}$$

Conditional on winning auction j, buyer's i expected surplus is:

$$\tilde{U}_i\left(v^i, r_{j,t}, b_j^{-i}\right) = r_{j,t} v^i - \mathbb{E}\left(b_j^{(2)} | b_j^i > \max_{-i}\left\{b_j^{-i}\right\}\right) \tag{1.7}$$

where $b_j^{(2)}$ is the second highest bid in j's auction and $\{b_j^{-i}\}$ is the set of all bids in seller's j auction j, excluding bidder i.

Equation (1.7) takes into account only the subset of buyers that have decided to bid in seller's j auction. The ex-ante expected surplus from bidding in seller's j auction is given by

$$U_i\left(v^i, r_{j,t}\right) = \sum_{n=0}^{N-1} P\left(n+1 \text{ bidders in auction } j\right) P\left(v^i = \max_k v^k\right) \times \left(r_{j,t} v^i - \mathbb{E}\left(b^{(2)} | b^{(1)} = b^i, n+1 \text{ bidders}\right)\right) \tag{1.8}$$

The first term in expression 1.8 is the probability that given that there are $n+1$ bidders in seller's j auction, bidder i has the highest valuation among all the bidders k in that auction. The second term stands for the expected instantaneous expected surplus of bidder i given that he wins the auction of seller j in which there are in total $n+1$ bidders.

Buyer's i expected surplus from bidding in seller's j auction increases both in seller's j reputation and in the expected distance between the highest and the second highest valuations in seller's j auction. Hence, buyers face a tradeoff: on the one hand, a buyer has a higher expected revenue from bidding in the auction of a seller with a relatively high reputation, since this implies a higher probability of receiving the good, conditional on winning the auction. On the other hand, given his valuation v^i, bidding in the auction of lower reputation sellers might bring a higher surplus and a higher probability of winning if buyers of relative low valuation bid in these auctions.

Beliefs Updating

Buyers are forming and updating their beliefs about the type of seller they are facing in a Bayesian fashion. Having observed the history of seller j consisting in the sequence of outcomes of past transactions up to time t, buyers hold *common* beliefs, $\mu_{j,t-1}$, at the beginning of the t-th period about seller j being of "good" type. After the realization of the outcome of period t, sellers and buyers of period $t+1$ revise their beliefs about other sellers' types using the Bayes rule. In case of a success, the beliefs at the beginning of period $t+1$ are update to $\mu_{j,t} = \mu^S_{j,t-1}$, while in the case of a failure the public believes that seller j is good with probability $\mu_{j,t} = \mu^F_{j,t-1}$. Concretely,

$$P_t\left(\theta_j = g \middle| y_{j,t} = S, \mu_{j,t-1}; \left\{\tilde{e}^{\theta_j}_{j,t}\right\}\right) \equiv \mu^S_{j,t-1} = \frac{\mu_{j,t-1} g \tilde{e}^g_{t,j}}{\mu_{j,t-1} g \tilde{e}^g_{j,t} + (1-\mu_{j,t-1})\, b \tilde{e}^b_{j,t}} \tag{1.9}$$

and

$$P_t\left(\theta_j = g \middle| y_{j,t} = F, \mu_{j,t-1}; \left\{\tilde{e}^{\theta_j}_{j,t}\right\}\right) \equiv \mu^F_{j,t-1} = \frac{\mu_{j,t-1}\left(1 - g\tilde{e}^g_{j,t}\right)}{\mu_{j,t-1}\left(1 - g\tilde{e}^g_{j,t}\right) + (1-\mu_{j,t-1})\left(1 - b\tilde{e}^b_{j,t}\right)} \tag{1.10}$$

1.3.3 Markov Strategies

As specified above, the public history of a seller records only the outcomes of past transactions. Such information restriction when describing strategies still entails many equilibrium in which reputation has different roles. In order to rule out non intuitive equilibria, we will focus on Markov strategies, i.e. we want strategies to make behavior by each seller at any t only dependent on a relatively small subset of the public information available, or on a sufficient statistic of it.[9]

Assumption 6 implies that, although sellers can observe past prices, they will turn out not to be payoff relevant variables at the time of deciding about effort.

In the case of on-line auction platforms such as eBay, the information available to sellers and potential buyers about each other is significantly limited. Usually, such information amounts to *feedback* scores (current and historical, up to a certain past date) which represent a summary statistic of past evaluations by buyers or sellers. This information is public, and is the *same* information observed by a researcher / economist / econometrician.

Given assumptions 1-6 and the above considerations, the reputation of seller j at the beginning of period (auction) t, $\mu_{j,t-1}$, is a *sufficient statistic* about that seller's type: it summarizes all the payoff-relevant information about seller's j type available in the only publicly observed variables y_j^t. Such information is payoff-relevant due to the fact that it helps predict the behavior of sellers in the current auction (recall that buyers are short lived in our benchmark game). Note that, given the finite-time nature of the game, then the state space will also include the time period t.

Given the above, a Markov strategy for seller j is :

$$e_{j,t} : [0,1]^2 \times \Theta \times T \longrightarrow [0,1]$$

where $e_{j,t}(\mu_1, \mu_2; \theta_j) \equiv e_{j,t}^{\theta_j}(\mu_1, \mu_2)$ is the effort choice of a seller with type θ_j at period t, when buyer's beliefs about all sellers' types are described by the vector $\{\mu_j\}_{j=1}^2 \in [0,1]^2$.

When facing sellers with different reputations, buyers' strategies consist in an *auction selection vector* and in a *set of bids*. Given that we restrict buyers to bid in only one auction, an auction selection strategy for buyer i is a mapping

$$s^i : [0,1]^2 \times [0,1] \longrightarrow [0,1]^2$$

where $s^i(v^i; \mu_1, \mu_2) \equiv \left\{ s_j^i\left(v^i; \mu_1, \mu_2\right) \right\}_{j=1}^2$ is a 2-dimensional vector of selection probabilities.

1.3.4 Timing

The timing is presented in figure 1.1. The game proceeds in the following way: at time $t = 0$, the game is in state $(\mu_{j,0})_{j \in \mathcal{J}}$, where $\mu_{j,0}$ $j \in \mathcal{J}$ are the publicly held time-0-beliefs (prior beliefs) about sellers being competent. We allow these beliefs to be

[9] Given the functional form for the conditional probability of success, $P(y = S|\theta, e) = \theta e$, then for any equilibrium, every possible history of delivery outcomes $h^{t-1} \in \mathcal{H}^{t-1}$ will have positive probability in such equilibrium. This will simplify considerably the equilibrium characterization, since there will be no "off-equilibrium path" information set.

different across sellers. Sellers open simultaneous auctions in which all sell the same homogeneous good. Buyers simultaneously select an auction and place a bid. The winners of each auction are announced and the second highest bid in each auction is paid out to the respective seller, as described in the previous section. Sellers decide then on the effort they want to put in delivering the object, bearing in mind that the probability of a success is proportional to their effort and that the success cannot be guaranteed. Any of the two possible outcomes - success $y_{j,0} = S$ or failure $y_{j,0} = F$ - will be then truthfully reported by buyers and enter the public record. Based on the outcome, other sellers and buyers update their beliefs about seller's j type. The infinite game consists of infinitely many repetitions of the stage-game described above.

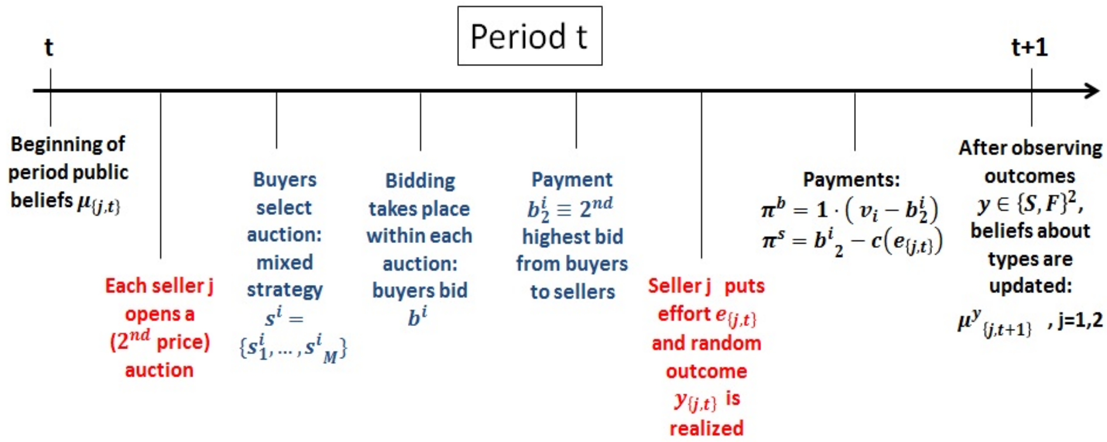


FIGURE 1.1: Timing

1.3.5 Equilibrium

Buyers' equilibrium behavior derived in Bapna, Dellarocas, and Rice, 2010 is based on their perceived success probability $r_{j,t}$. In our setup, buyers observe sellers' reputation $\mu_{j,t-1}$ and, anticipating sellers' equilibrium behavior $\tilde{e}^{\theta_j}_{j,t}$, buyers form a measure of the success probability $r_{j,t}$for each seller, as described in equation 1.5. Note that the buyer's problem and equilibrium behavior can be characterized for a *given set of beliefs*. In other words, buyers play a one shot game and their behavior is static by assumption.[10]

Definition 7 *A tuple* $\left(\left\{e^g_j(\boldsymbol{\mu}), e^b_j(\boldsymbol{\mu})\right\}_j, \left\{s^i(v^i;\boldsymbol{\mu}), b^i(v^i;\boldsymbol{\mu})\right\}_i, \boldsymbol{\mu}\right)$*, with* $\boldsymbol{\mu} \equiv \{\mu_1, \dots\mu_J\}$*, is a* ***Markov Perfect (Bayesian) Equilibrium*** *(MPE) of the reputation game with asymmetric information if, for a buyer's valuation distribution* $F(\cdot)$*:*

1. *Buyers selection strategies* $s^i(v^i;\boldsymbol{\mu})$ *and bidding behavior* $b^i(v^i;\boldsymbol{\mu})$ *are optimal given* $\boldsymbol{\mu}$*, their valuation* v^i *and distribution* F*.*

2. *Sellers choose effort* $\left\{e^g_j(\boldsymbol{\mu}), e^b_j(\boldsymbol{\mu})\right\}_j$ *to maximize* ***continuation*** *payoffs* (1.4)*, given* $\boldsymbol{\mu}$*.*

[10]We analyze the role of assuming short lived buyers in the robustness section.

3. *Sellers' behavior is consistent with the* ***Markov-belief function*** (1.5)*:*

$$r_{j,t} = \mu_{j,t-1} \cdot ge^{g}_{j,t} + (1 - \mu_{j,t-1}) \cdot be^{b}_{j,t}$$

4. *Beliefs are updated according to Bayes' rule:*

$$\mu_{j,t}(\theta_j = g|, y_t, \mu_{j,t-1}) = \frac{P(y_{j,t}|\theta_j = g) \cdot \mu_{t-1}}{\sum_{\theta_j} P(y_{j,t}|\theta_j) \cdot P(\theta_j)} \quad \forall y_{j,t} \; if \; \sum_{\theta_j} P(y_{j,t}|\theta) \cdot P(\theta_j) > 0.$$

1.4 Characterization and Analysis

In order to disentangle the effects that competition and reputation incentives have on effort an welfare, we study separately two versions of the game: (i) the case when there only one seller (and everyone knows so) , and (ii) the case with 2 sellers.

1.4.1 Reputation Effects with One Seller

We first analyze the two period game with a *single* seller. This will allow us to isolate how the two main information asymmetries shape the equilibrium effort strategies and reputation building. We do this by characterizing the market equilibrium in such scenario and then comparing with three alternative informational assumptions: (i) a situation in which the seller can *commit ex-ante* (i.e. before buyers bid) to exert some effort, and where the type (ability) of the seller is observed; (ii) a second scenario where the seller can commit, but her type is now unobserved; and (iii) our case of interest, in which the seller cannot commit ex-ante, and her type is unobserved, i.e., where both adverse selection and moral hazard problems are present.

Our results are consistent with a (finite horizon version) of Holmstrom, 1999 and Mailath and Samuelson, 2001: there is an equilibrium (which in our case is unique in Markov strategies) in which the monopolist *temporally* exerts costly effort, and where such effort strategy is non-monotone in initial beliefs (the state variable).

Building a Reputation in The First Period

A seller auctions off a homogeneous good over two periods. Given the timing in the game, the seller will employ the maximal costless effort $\underline{e}$ in period 2 as there are no future payoffs from which he can gain when employing a costly effort, independently of her type and on the outcome of the first period.

Since there are no alternative auctions, buyer's selection strategy is trivial. Equilibrium bidding implies that the expected revenue of the seller in a second price auction in which valuations are drawn from an uniform distribution and the total number of buyers is N equals:

$$E\left(b_t^{(2)}\right) = r_t \times H_2(N) \tag{1.11}$$

where $E\left(b_t^{(2)}\right)$ is the expected second highest *bid*, and $H_2(N) \equiv \mathbb{E}\left(v^{(2)}|N \text{ bidders}\right)$ is the expected *valuation* of the second highest bidder (a second order statistic; see appendix A.2 for the derivation of the functional form for $H_2(N)$).

There are two important features of $H_2(N)$: (i) it is a function of N, the number of buyers each period; and (ii) it does not depend on the seller's reputation, a feature that will not be necessarily true once we add competition between sellers, and which will drive seller's strategic behavior. From now on, we omit the explicit dependence on N. Equation (1.11) therefore implies that the expected second highest bid will be a linear function in monopolist's reputation.

Upon receiving the money, seller 1 decides how much effort to put in delivering the item. The instantaneous expected payoff of the monopolist at the beginning of period t, conditional on her type θ is given by:

$$\pi_t(\theta, e_t^\theta, \tilde{e}_t^\theta, \mu_t) = E\left(b_t^{(2)} \mid \mu_t\right) - c(e_t^\theta) \tag{1.12}$$

The present value (PV) of a seller's profits conditioning on her type θ is then

$$\Pi_0^{monop}\left(\theta, e_1^\theta, \tilde{e}_1^\theta, \mu_0\right) = \pi_1(\theta, e_1^\theta, \tilde{e}_1^\theta, \mu_0) + \delta\mathbb{E}\left[\pi_2\left(\theta, \underline{e}, \mu_1\right) \middle| \theta, e_1^\theta\right] \tag{1.13}$$

where

$$\mathbb{E}\left[\pi_2\left(\cdot\right) \middle| \theta, e_1^\theta\right] = e_1^\theta\theta \cdot \pi_2(\theta, e_2^\theta, \tilde{e}_2^\theta, \mu_1|y_0 = S) + \left(1 - e_1^\theta\theta\right) \cdot \pi_2(\theta, e_2^\theta, \tilde{e}_2^\theta, \mu_1|y_0 = F) \tag{1.14}$$

Equilibrium effort levels for each type of seller have to solve

$$\max_{e_1^\theta} \ \Pi_0^{monop}\left(\theta, e_1^\theta, \tilde{e}_1^\theta, \mu_0\right) \tag{1.15}$$
$$s.t. \quad e_t^\theta \in [\underline{e}, 1]$$

Best responses for each type of seller are characterized by the system of first order conditions for $\theta = \{g, b\}$:

$$c'(e_1^{*\theta}) = \delta\theta\left(\pi_2^S - \pi_2^F\right) = \delta\theta H_2(N)\left(\mu_1^S - \mu_1^F\right)(g-b)\underline{e} \tag{1.16}$$

where, as before, $H_2(N)$ is the expected second highest value in the seller's auction and $e_t^{*\theta}$ is the best reply of seller-type θ to the effort of seller-type $-\theta$. These are, in other words, optimal effort levels employed in period $t = 1$).

Lemma 1 *The single crossing condition yielded by the complementarity of effort and types, implies that higher types always choose higher efforts:* $e_1^{*g} > e_1^{*b}$.

Proof. Note that

$$c'(e_1^{*g}) = \delta g H_2(\mu_1^S - \mu_1^F)(g-b)\underline{e} > \delta b H_2(\mu_1^S - \mu_1^F)(g-b)\underline{e} = c'(e_1^{*b})$$

The cost function being monotonically increasing and convex results in $c'(e_1^{*g}) > c'(e_1^{*b})$, from which follows that $e_1^{*g} > e_2^{*b}$

With unobservable types, the *reputation* attributed to the seller at time $t = 2$ crucially depends on the outcome of the first period: $r_2 = (\mu_1 g + (1-\mu_1)b)\underline{e}$, where μ_1 are updated beliefs given the outcome of the first period ($\mu_1 = \mu_1^S$ or $\mu_1 = \mu_1^F$). The buyers active at this stage will bid their expected valuation for the good: $b^{*i}(v^i) = r_2 v^i$. Given that reputation r_2 is increasing in beliefs $\frac{\partial r_2}{\partial \mu_1} = (g-b)\underline{e} > 0$, a monopolist has high incentives to conclude period 1 with a success as $\mu_1^S > \mu_1^F$. Therefore, employing a higher but costly effort level in period 1 will raise the probability of a success and the implied future payoffs. In determining the optimal effort level, the monopolist will trade off the future benefits of a higher reputation with the present cost of effort.

Proposition 1 below establishes the existence and uniqueness of an equilibrium in the 2 periods game, in which the monopolist seller temporally exerts positive effort. In addition, proposition 2 characterizes the role of a reputation mechanism in such equilibrium.

Proposition 1 *An equilibrium of the two-periods game where a monopolist of unobservable type chooses (temporally) costly effort exists and is unique.*

Proof. See appendix A.4

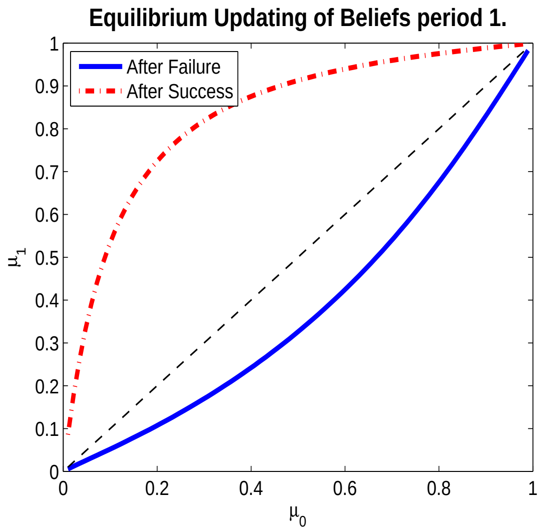
Proposition 2 below characterizes the equilibrium with moral hazard and adverse selection

Proposition 2 *For any type* θ, $\theta \in \{b, g\}$, *in the Markov equilibrium with hidden effort and unobserved type:*

1. *For any* $\mu \in [0,1]$ *the equilibrium effort level in period 1 increases with the discount factor* δ; *i.e.,* $\frac{\partial e_{j,1}^{\theta}(\mu)}{\partial \delta} > 0, \forall j, \theta$.

2. *For any* $\mu \in [0,1]$ *the effort level of the good type in period 1 is increasing in* g *and decreasing in* b; *i.e.,* $\frac{\partial e_{j,1}^{\theta}(\mu)}{\partial g} > 0$ *and* $\frac{\partial e_{j,1}^{\theta}(\mu)}{\partial b} < 0$. *The opposite is true for the effort level of the bad type.*

3. *The equilibrium effort level in period 1 is non-monotone and concave in initial beliefs* μ_0: $\frac{\partial e_{j,1}^{\theta}(\mu)}{\partial \mu_0} > 0$ *for* $\mu_0 < \mu^*$ *and* $\frac{\partial e_{j,1}^{\theta}(\mu)}{\partial \mu_0} < 0$ *for* $\mu_0 > \mu^*$. *The threshold* μ^* *is a function of the different parameters of the game, and in particular, of* g *and* b.



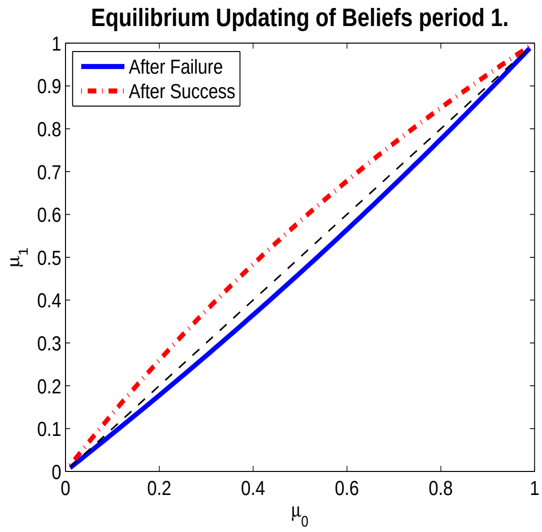


FIGURE 1.2: Equilibrium updating of beliefs after S and F. Left: $g = 0.9$ $b = 0.1$
Right: $g = 0.7$ $b = 0.5$

Proof. See appendix A.4.

Result 1 in proposition 2 is straightforward. The higher the discount factor for a seller, the more relevant future revenues become for her, and therefore reputation building has a higher value. Result 2. captures an important aspect of on-line markets (and markets in general with imperfect information in a dynamic setting). Figure 1.2 presents the argument graphically: the plot shows the updating of type-beliefs, for an initial prior, after a successful transaction and after a failed one (equations (1.9) and (1.9)) . The left-hand side plot presents the case where $g = .9$ and $b = .1$, while the right-hand side plot presents the case of $g = .7$ and $b = .6$.: if the difference $g - b$ is close to 1, we are in a situation in which a seller can be either very able / willing, or almost not able at all, to deliver successfully. For any initial belief, a seller of good type has then a strong incentive to separate herself from a seller of bad type through relatively high levels of effort, the reason being that a successful outcome will reveal a lot of information about the sellers ability; clearly, this is true as long as this marginal benefit is higher than the marginal cost of effort. For a bad type seller (i.e. the incompetent seller) things are different. Although higher g creates incentives for her to try to mimic a good seller, the same happens if the value of b increases. The reason is that when $g - b$ is small and effort is not observable, it is harder for buyers to distinguish among sellers of different abilities. This implies that effort will pay relatively more to bad type sellers. The convexity of the cost function then drives the low level of efforts in equilibrium. This is the standard likelihood ratio property coming into play.[11]

Result 3. requires a careful explanation. The upper row in figure 1.5 depicts how the optimal effort for both types varies with different initial beliefs. As long as the seller cannot commit ex-ante to an effort level, the uncertainty about the seller's type is

[11]See appendix A.3 for a more detailed description of this.

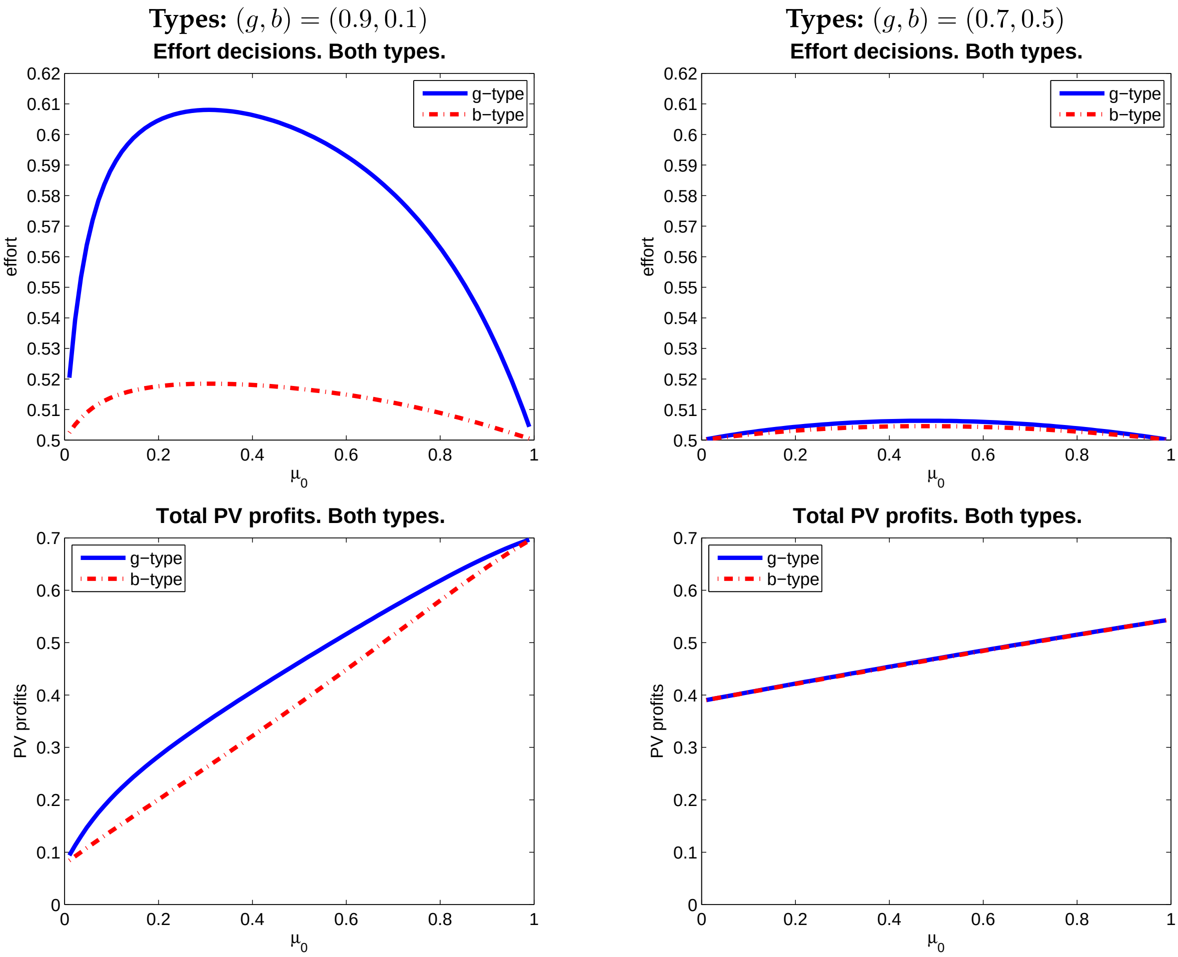


FIGURE 1.3: Equilibrium effort (top) and PV profits (bottom). Left: $g = 0.9$ $b = 0.1$ Right: $g = 0.7$ $b = 0.6$

what drives her incentives of employing costly effort. As the buyers believe with more certainty that they are facing a particular type the game starts resembling the one with observable types described above. It follows that the incentives for any seller type to exert costly effort vanishes. This is similar to the findings in Holmstrom, 1999. In our game, this mechanism is clear from the optimality condition (1.16) for each type. As $\mu_0 \to 1$ also $\mu_0^S \to 1$ and $\mu_0^F \to 1$ so $\mu_0^S - \mu_0^F \to 0$. The equilibrium effort level for each type will converge then to the maximal costless effort level $e^{*\theta} \to \underline{e}$. The same principle applies for the case where $\mu_0 \to 0$. The non-monotonicity in μ is close to the concept of "spending" a reputation described in Mailath and Samuelson, 2001.

Alternative Information Assumptions

If the seller's type is observable, the game is trivial. The only possible equilibrium is one in which the monopolist exerts the maximum costless effort in both periods, i.e. $e_t^{\theta obsNC} = \underline{e}$, $t = 1, 2$, $\theta \in \{b, g\}$. Expected profits for the seller are then given by

$$\Pi^{\theta obsNC} = (1 + \delta)\left(\underline{e}\theta H_2\right) \tag{1.17}$$

This result is straightforward: if the ability of the seller is publicly observable, the seller has no "reputation" to defend and maintain. The perceived probability of success of a transaction at time t is given by $r_t = \theta \underline{e}$,$t = 1, 2$, $\theta \in \{b, g\}$. The corresponding bid in the first period by buyer i will be $b(v^i) = \theta \underline{e} v^i$. Furthermore, the outcome of the first period is irrelevant for the second period since beliefs are degenerate. Employing costly effort has then no effect on future payoffs, and although they do affect the equilibrium bids in the first period (since buyers predict the seller's behavior), the seller is not able to commit to a given effort ex-ante. This implies an equilibrium effort that satisfies $c(e_t^{\theta obsNC}) = 0$. Independently of what had happened in period 1, the seller will always employ the maximal costless effort in period 2. Given the monopolist's type and anticipating her defective behavior, a buyer with valuation v^i will submit the following bid in the second period $b(v^i) = \theta \underline{e} v^i$.

Efficient Effort

The efficient effort can be defined, given the buyer that has won the auction (the buyer with highest valuation $v^{(1)}$) , as a solution to the maximization of the total (ex-ante) surplus $\mathcal{W}_t$ of the transaction[12]:

$$\max_{\{e_1^\theta\}_\theta} \mathcal{W}_1 = -c\left(e_1^\theta\right) + \theta_1 e_1^\theta \mathbb{E}\left(v^{(1)}\right) \tag{1.18}$$

s.t.

$$e_1^\theta \in [0, 1]$$

where $\mathbb{E}\left(v^{(1)}\right)$ is the expected highest valuation. The first order conditions of such problem for $\theta = b, g$ are

$$c'\left(e_1^{\theta eff}\right) = \theta \mathbb{E}\left(v^{(1)}\right) \tag{1.19}$$

In other words, for a given buyer's valuation $v^{(1)}$, the efficient allocation is such that a seller of good type should exert higher effort than a seller of bad type. In addition, (1.19) implies that, given the properties of $c(\cdot)$ and that $v_i \sim [0, 1]$, $e_1^\theta > \underline{e}$. In other words, the efficient effort is greater than the maximal costless effort. The idea is simple: in our model, the seller does not value the good, while the buyer's value of the good is v. Given that the price payed is just a transfer between the buyer and the seller, efficiency stipulates that effort needs to compensate the marginal cost with a positive impact on the delivery technology.

Commitment and Observable Type

Consider first the case where a seller can credibly commit to a certain level of effort *before* the buyers bidding decision, and where her type is publicly observed / known.

[12]Recall that when there is just one seller all buyers bid in her auction; therefore, selection plays no role, which implies that we donot care about efficient allocation of buyers to sellers.

In such scenario, beliefs would play no role, and the probability of a success becomes $r_t = \theta e_t$. This implies that the problem for a seller is now a static one. Concretely, she would solve

$$\max_{e\in[0,1]} \theta e H_2 - c(e)$$

The FOC for such problem is

$$c'\left(e^{\theta obsC}\right) = \theta H_2 \tag{1.20}$$

The ability to commit allows the seller to shift her focus towards the *current* period: she now cares about the effect of her effort on the strategies of current buyers, and therefore drives the effort away from the maximal costless effort. However, note that $H_2 < \mathbb{E}\left(v^{(1)}\right)$: since the allocation mechanism is a 2nd price auction, the expected revenue for the seller incorporates the expected 2nd highest bid, which implies that $e^{\theta obsC} < e^{\theta eff}$, i.e. the effort decision under commitment and observable type would be lower than the efficient one.

What about seller's profits? For a seller of type θ, these are given by

$$\Pi^{\theta obsC} = (1+\delta)\left(e^{\theta obsC}\theta H_2 - c\left(e^{\theta obsC}\right)\right) \tag{1.21}$$

Lemma 2 *For any $\theta \in (0,1)$, $\Pi^{\theta obsNC} < \Pi^{\theta obsC}$. In addition, $\Pi\left(e^{\theta obsC}\right) > \Pi\left(e^{\theta eff}\right)$: the seller makes higher profits than if she would employ the efficient effort computed in (1.19). Total surplus in the efficient effort case, however, will be* higher *than under the commitment & observable type case.*

Commitment and Unobservable Type

Now we assume that the seller can still credibly commit to a certain level of effort *before* the buyers bidding decision, but the type is unobserved. This is plainly a signaling game between the seller and current and future buyers.

Lemma 3 *In this signaling game where a seller can commit to some effort level before the auction starts, a separating equilibrium doesn't exist. A pooling equilibrium in which both types of seller announce the same effort decision in both periods always exists. The maximal effort levels to which a seller of type θ can commit in both periods depend on the initial reputation μ_0 and on the number of buyers N.*

Proof. See appendix A.4.

The reasoning is the following: assume a separating equilibrium where a seller of type g announces e^g and a seller of type b announces e^b, and where $e^g < e^b$ w.l.o.g. Types would then be revealed right after the announcement, which would imply $\mu_1 = \mu_2 = 1$ if $e = e^g$, or $\mu_1 = \mu_2 = 0$ if $e = e^b$.

1.4.2 Strategic Competition for Heterogeneous Buyers

Understanding the fierceness of competition in the future is relevant in order to decide what kind of reputation building is the seller willing to embark herself in. In this section, we investigate the role of such mechanism in understanding the equilibrium behavior of buyers and sellers, as well as its potential to explain observed outcomes.

A buyer interested in the good can observe the reputation generated from past data for *all* the available sellers of that particular product. As it was mentioned before, a seller observes this very *same* information about other sellers who are competing against her. If she believes that buyers select a seller taking into account the reputational information, then other sellers reputations become payoff relevant when deciding about effort. Current beliefs will help to predict the competitor's behavior and therefore the strength of competition faced *next* period, given the updating of reputation. Sellers then start to compete *strategically*, understanding the effect of their actions not only in their absolute reputation value, but also in their "attractiveness" relative to other sellers.

Conditional on her type, and given the information that buyers consider when selecting an auction and bidding, seller j instantaneous profits in period $t = 1, 2$ are:

$$\pi_{j,t}\left(e_{j,t}^{\theta_j}, \tilde{e}_{j,t}^{\theta_j}, \tilde{e}_{-j,t}^{\theta_{-j}}, \mu_{j,t-1}, \mu_{-j,t-1}\right) = \mathbf{R}_{j,t}\left(r_{j,t}, r_{-j,t}\right) - c\left(e_{j,t}^{\theta}\right)$$

where $r_{j,t}$ was defined in equation (1.5), as a function of $e_j^{\theta_j}$ and μ_j. $\mathbf{R}_{j,t}\left(r_{j,t}, r_{-j,t}\right)$ is the expected revenue of seller j at time t. The expected revenue for seller j given his and his competitor's reputation is simply the expected second highest bid, given by

$$\mathbf{R}_{j,t}\left(r_{j,t}, r_{-j,t}\right) \equiv \mathbb{E}\left(b_{j,t}^{(2)}\right) = r_{j,t} H_{2j}\left(r_{j,t}, r_{-j,t}\right) \tag{1.22}$$

where, as before, $H_{2j}\left(r_{j,t}, r_{-j,t}\right) \equiv \mathbb{E}\left(v^{(2)}\right)$ is the expected second highest valuation in seller j's auction, given the perceived probability of a successful delivery by each seller in the current period. Expression (1.22) makes clear that now seller j's reputation affects her current revenues through an additional channel relative to the one seller case: the *expected valuation* of the buyers, as well as the *number* of buyers, that will decide to bid in seller j's auction, summarized by $H_{2j}\left(r_{j,t}, r_{-j,t}\right)$.

Expected 2^{nd} Highest Valuation in Seller j's Auction

In our setting, when there is just one seller in the market, such seller receives the same expected type of buyers irrespective of her reputation (although, as we argued, the amount that each buyer will bid does depend on the reputation). When two sellers compete, buyers need to select and bid taking into account the relative reputations. $H_{2j}(\cdot, \cdot)$, both in the first and in the second period, will now emerge *endogenously* from the auction selection and bidding equilibrium. Here we present the main features of the selection equilibrium; a more detailed description of H_{2j} is presented in appendix A.2.

A buyer with private valuation v^i who has the chance to bid in only one period and one auction faces the following trade-off: on the one hand bidding in the auction of the seller with the highest reputation results in a higher expected revenue upon winning but it also entails a lower probability of winning the auction. On the other hand, winning the auction of a seller with a relative lower reputation is more likely but also the expected revenue upon winning that auction is lower. This trade-off is clear from the expression (1.8) for the expected surplus that a buyer receives from bidding on seller j's auction. Following Bapna, Dellarocas, and Rice, 2010, if we restrict the analysis to symmetric selection strategies which are piecewise continuous on a buyer's type, this trade-off generates a selection equilibrium with three main features that we describe in proposition 3 below.

Proposition 3 ***One-bid selection equilibria.*** *Assume, without loss of generality, that $r_j \leq r_{-j}$. When we restrict selection strategies to be piecewise continuous in buyer's type, then:*

1. *There is no pure strategy selection equilibria.*
2. *Buyers are divided into 2 type zones, with a zone threshold t_v^*:*

$$t_v^* \left(r_j, r_{-j}\right) = \sqrt[N-1]{\frac{r_j}{r_{-j}}} \tag{1.23}$$

3. *Buyers with type $v^i < t^*$ choose a selection strategy*

$$s^i \left(r_j, r_{-j}\right) = \left(s_j^i, s_{-j}^i\right) = \left(\frac{\sqrt[N-1]{r_{-j}}}{\sqrt[N-1]{r_j} + \sqrt[N-1]{r_{-j}}}, \frac{\sqrt[N-1]{r_j}}{\sqrt[N-1]{r_j} + \sqrt[N-1]{r_{-j}}}\right) \tag{1.24}$$

while buyers with type $v^i \geq t^*$ choose a selection strategy

$$s^i \left(r_j, r_{-j}\right) = \left(s_j^i, s_{-j}^i\right) = (0, 1) \tag{1.25}$$

where s_j^i is the probability that buyer i assgns to bidding in auction (seller) j.

Proof. See appendix A.4.

Proposition 3 is a straightforward extrapolation of lemma 2 and proposition 2 in Bapna, Dellarocas, and Rice, 2010 to our setting. The assumptions that allows us to do this in our case are two: (i) the fact that we are focusing on short lived buyers, (ii) that use Markov strategies, mappings from the space of reputations to the space of actions. Buyers auction selection strategy depends on the perceived probability of successful delivery for all sellers, $r_{j,t}$ and $r_{-j,t}$. These probabilities are a function of reputation $(\mu_{j,t-1})_{j=1}^2$, effort decisions in equilibrium and the total number of bidders N. The equilibrium behavior, represented by the expected second highest bid in seller j's auction, then point out at a form of probabilistic assortative matching: buyers self-select into 2 type zones. Those in the lower type zone will mix between the two sellers, whith selection probabilities being inversely proportional to the "trustworthiness" (i.e. r) of an auction. Those on the upper type-zone bid with certainty in the more trustworthy seller.

First Period Competition Through Reputation Building

As in the one seller case, effort in the second period for seller j has no value ;therefore, minimum effort is still optimal: $e^{\theta_j}_{j,2} = \underline{\mathrm{e}} \ \forall \ j, \theta_j$.

In the first period, conditional on the initial reputations $(\mu_{j,0}, \mu_{-j,0})$, each seller j of type θ is faced with the following problem

$$\max_{e^{\theta_j}_{j,1}} \left\{ r_{j,1} H_{2,j}\left(r_{j,1}, r_{-j,1}\right) - c\left(e^{\theta_j}_{j,1}\right) + \delta \mathbb{E}\Big(r_{j,2} H_{2,j}\left(r_{j,2}, r_{-j,2}\right) - c\left(\underline{\mathrm{e}}\right) \Big| e^{\theta_j}_{j,1}, \theta_j \Big) \right\} \tag{1.26}$$

s.t.

$$e^{\theta_j}_{j,1} \in [\underline{\mathrm{e}}, 1]$$

Here, $r_{j,t} = P\left(y_{j,t} = S | \boldsymbol{\mu_{t-1}}\right)$ is the probability of success attributed to seller j at the beginning of period t.

Given current beliefs $\boldsymbol{\mu}_t$, beliefs in $t+1$ will depend on $y_{j,t}$ and the predicted effort for each type of seller. As in previous sections, we denote $\mu^S_j\left(e_j\right)$ and $\mu^F_j\left(e_j\right)$ the updated beliefs after $y_{j,t} = S$ and $y_{j,t} = F$ respectively. Denote seller's j profit in period 2 as $\pi^{y_{j,1}y_{-j,1}}_{j,2}$, conditional on his and his competitor outcome of the first period $y_{j,1}$ and $y_{-j,1}$. In addition, we denote $r^{y_{j,1}}_{j,2}$ the anticipated probability of success of seller j in the second period, given the outcome of his transaction in the first period $y_{j,1}$: $r^{y_{j,1}}_{j,2} = \left(\mu^{y_{j,1}}_{j,1} g\tilde{e}^g_{j,2} + \left(1 - \mu^{y_{j,1}}_{1,1}\right) b\tilde{e}^b_{j,2}\right)$.

Given the (conditional) independence of outcomes, the expected profit of seller j at time 2, given her type, effort and perceived effort from her competitor is

$$\begin{aligned} E\left[\pi_{j,2} | e^{\theta_j}_{j,1}, \theta_j\right] &= e^{\theta_j}_{j,1} \theta r_{-j,1} \pi^{SS}_{j,2} + e^{\theta_j}_{j,1} \theta_j \left(1 - r_{-j,1}\right) \pi^{SF}_{j,2} \\ &+ \left(1 - e^{\theta_j}_{j,1} \theta_j\right) r_{j,1} \pi^{FS}_{j,2} + \left(1 - e^{\theta_j}_{j,1} \theta_j\right) \left(1 - r_{-j,1}\right) \pi^{FF}_{j,2} \end{aligned}$$

Again, effort best replies to all the other seller's effort (or to their current reputation) are characterized by the FOCs of the problem:

$$c'\left(e^{\theta_j}_{j,1}\right) = \theta_j \delta \Big(r_{-j,2}\left[\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right] + \left(1 - r_{-j,1}\right)\left[\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right] \Big) \tag{1.27}$$

When deciding about effort after auctions in the first period have closed, the timing and lack of commitment from sellers implies that they will only compete in *reputations* for future revenues $\boldsymbol{R}_2$. In other words, an announcement to a certain effort level is ruled out by a typical time inconsistency problem. Proposition 3 reveals why the revenues in period 2 for seller j are a function period-2 buyers' beliefs about the probability of success of both sellers; these probabilities, in turn, arise through the updating of reputation after the effort decisions and outcome in period 1. Therefore, seller's j strategic behavior will weight the current cost of effort against the (discounted) potential future revenue reaps from her competitor; this is the RHS term between parenthesis in eq. (1.27).

In equilibrium, revenue reaps will be determined by the interaction of three components: (i) the updating function; (ii) the valuation threshold $t_v^*(r_1, r_2)$; and (iii) the bidding probabilities $s_j^i(r_j, r_{-j})$. The current public information about her type and her competitors type is therefore payoff relevant for a seller since it helps predict her competitor's action (effort) in the current period, which is also targeted at influencing buyer's beliefs next period. More precisely, the reputations at the beginning of period 1 (i.e. $\boldsymbol{\mu}_0$) allows sellers to understand the marginal effect of effort when there is competition.

Before characterizing the equilibrium(s), we show that the primitives of the game generate effort decisions that are monotonic in types, or what is usually known as single-crossing condition within a revealed preference argument. We state this in Lemma 4 below.

Lemma 4 Single-Crossing condition*: Let* $e_{j,1}^{\theta_j} : [0,1]^2 \times \Theta \longrightarrow [0,1]$ *be an effort strategy for seller* $j = 1, 2$*, as a function of her type* $\theta_j \in \Theta$ *and reputations* $\boldsymbol{\mu} \equiv \{\mu_j, \mu_{-j}\}$*. Then, given the multiplicative technology* $P(y_j = S|\theta, e) = \theta e$*, it is the case in equilibrium that* $e_j^g(\boldsymbol{\mu}) \geq e_j^b(\boldsymbol{\mu})$, $\forall \boldsymbol{\mu} \in [0,1]^2$, $\forall j$.

Proof. See appendix A.4.

It turns out that, similar to the one seller case, there is a unique equilibrium in (non-stationary) Markov strategies. This equilibrium has two interesting (and somewhat counterintuitive) properties regarding the comparative statics on a competitor's prior reputation and the total number of sellers in the market. Propositions 4 and 5 below state these features rigorously.

Proposition 4 *An equilibrium of the two periods game in which two sellers of unobservable are competing for heterogenous buyers exists and is unique.*

Proof. See appendix A.4.

Proposition 5 ***Characterization of the equilibrium*** *In the Markov equilibrium where two sellers compete during two periods through second price auctions,*

1. *Seller j's ($j = 1, 2$) effort decision in period 1 is still non-monotonic in her own current (prior) reputation $\mu_{j,0}$, for $\theta \in \{g, b\}$: $\frac{\partial e^{\theta}_{j,1}(\mu_{j,0},\mu_{-j,0})}{\partial \mu_{j,0}} > 0$ for $\mu_{j,0} < \mu^*$ and $\frac{\partial e^{\theta}_{j,1}(\mu_{j,0},\mu_{-j,0})}{\partial \mu_{j,0}} < 0$ for $\mu_{j,0} > \mu^*$. The threshold μ^* is a function of $\mu_{-j,0}$ and the different parameters of the game (in particular, of g and b).*

2. *For any $\left(\mu_{j,0}, \mu_{-j,0}\right) \in [0, 1]^2$ and type $\theta \in \{g, b\}$, seller j's effort decision in period 1 is monotonically decreasing in her competitor's prior reputation $\mu_{-j,0}$ ("intensity" of competition): $\frac{\partial e^{\theta}_{j,1}(\mu_{j,0},\mu_{-j,0})}{\partial \mu_{-j,0}} < 0$.*

3. *An increase in the total number of buyers in the market from N to N', with $N' > N$ and $N, N' \geq 2$ has two effects:*

   *(a) It **reduces the impact** of competitor's period-1-reputation, $\mu_{-j,0}$, on the incentives to build one's own reputation: $\left|\frac{\partial e^{\theta}_{j,1}(\mu_{j,0},\mu_{-j,0};N)}{\partial \mu_{-j,0}}\right| > \left|\frac{\partial e^{\theta}_{j,1}(\mu_{j,0},\mu_{-j,0};N')}{\partial \mu_{-j,0}}\right|$; and*

   *(b) for any $\left(\mu_{j,0}, \mu_{-j,0}\right) \in [0, 1]^2$, it **increases** the equilibrium effort **level** of both types of seller j, $j = 1, 2$:*
   $e^{\theta}_{j,1}\left(\mu_{j,0}, \mu_{-j,0}; N\right) < e^{\theta}_{j,1}\left(\mu_{j,0}, \mu_{-j,0}; N'\right)$ *for $\theta \in \{g, b\}$.*

Proof. See appendix A.4.

Discussion

Proposition 5 presents three interesting comparative statics results related to the impact of period-1 *strategic competition*, on reputation building. The first result states that, for a *fixed* prior belief about the competitor's type, effort in period 1 is non-monotone and concave in a seller's own prior reputation. This implies that the reputation mechanism discussed for the case of one seller is still in place. The second and more important result shows that a higher prior (current) reputation of a seller's competitor leads the seller to employ lower effort c.p. Third, there are two channels through which the size of the market influences sellers' behavior.

Below, we discuss the latter two features, since the first one has been already discussed for the case of 1 seller. In order to explain the intuition behind these, we will refer to figures 1.5, 1.4 and figure 1.6, in which we present, respectively, the first period equilibrium effort, the behavior of the equilibrium valuation threshold $t^*_v\left(r_j, r_{-j}\right)$, and the expected second highest valuation in a seller's auction for any pair of reputation values of the two sellers.

Comparative statics result 2 is rather counterintuitive. Figure 1.5 plots period-1 effort levels and profits; it considers the case in which sellers can be "very trustworthy" or "barely trustworthy" (i.e. $g = 0.9$ and $b = 0.1$), both for a small and a large number of bidders ($N = 5$ in the left column and $N = 50$ in the right column). The first row presents seller 1 effort levels as a function of her own reputation, for a fixed value of seller reputation; the second row plots seller 1 effort as a function of competitor reputation, for a fixed value of her own reputation. One the one hand, when introducing even minimal competition (i.e. adding a seller with a minimal reputation, but maintaining the number of buyers fixed) the expected level of revenues for a seller of any given reputation decreases, since as the pool of buyers that will self-select into the seller's auction is now (weakly) smaller. This is not surprising, but it is not what drives *marginal* effort decisions under competition. On the other hand, one would expect that the introduction of competition would, for certain regions of the relative reputation space, rise the *marginal* benefits from exerting effort. When the reputation of the competitor increases, effort increases would intuitively be generating (with higher probability) not only a better absolute reputation next period but also a base of bidders with better valuations, counterbalancing the improved competitor's reputation[13]. However, the opposite seems to be happening: when the prior reputation of a seller's competitor increases, the incentives for the seller to exert effort *decrease*.

FIGURE 1.5 ABOUT HERE

The explanation behind such decreasing incentives as the opponent's reputation improves, is to be found in how these help a seller predict her competitor's effort and its effect on future relative reputations. Ultimately, the latter will determine, through buyers behavior, period-2 revenues.

Recall that in the second period, effort is minimum: $e^{\theta}_{j,2} = \underline{e}\ \forall j, \theta$. Therefore, the perceived probability of a successful delivery $r_{j,2}$ will be completely determined from the updating of reputation, $\mu^{y}_{j,2}$. Given the pair of success probabilities, the behavior of the threshold valuation $t^{*}_{v}\left(r_j, r_{-j}\right)$ and the bidding probabilities $s^{i}\left(r_j, r_{-j}\right)$ as defined in proposition 3 determine the behavior of revenues for both sellers. Figure 1.4 plots the valuation threshold t^{*} as a function of seller 1 success probability r_1, for two values of r_2 (blue and green lines). When seller 1 has relatively low reputation in the last period (to the left of the dashed lines in both plots), an increase in her reputation increases t^{*} towards 1 (where $r_1 = r_2$). More importantly, an increase in the competitor's reputation (making seller 1 relative reputation even smaller) is translated into a *decrease* in the slope of $t^{*}_{v}\left(r_1, r_2\right)$. This implies a *smaller marginal return* of her own reputation, measured in terms of the type of buyers that will decide to bid in her auction. A similar behavior occurs when $r_1 > r_2$, i..e., when seller 1 has a relatively high reputation. For this region, an increase in r_1 implies that $t^{*}_{v}\left(r_1, r_2\right)$ will decrease and fewer buyers will decide to bid with probability >0 in seller 2, although the slope increases (decreases in absolute value) for higher values of r_2. These characteristics of t^{*} are the kernel of the instantaneous revenues of a seller (expected second highest bid).

The above properties from the selection equilibrium, combined with the way in which reputations are updated, shape the strategic competition in the first period. In the third row of figure 1.5 we plot the expected revenue function in equilibrium (incorporating equilibrium effort decisions predicted by buyers; the key characteristic is

[13]This analysis obviously does not take into consideration entry or exit decisions.

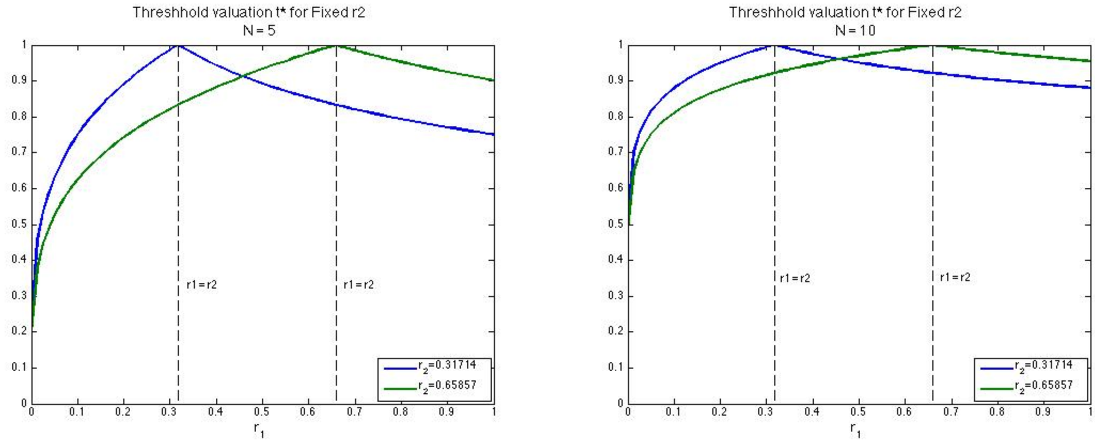


FIGURE 1.4: Equilibrium behavior of $t_v^*(r_j, r_{-j})$, $N = 5$ (left) and $N = 15$ (right). Fixed values of r_{-j}

that such function it exhibits decreasing differences. The main implication is that a seller's marginal expected revenue increases when his competitor's reputation $\mu_{-j,0}$, and therefore $r_{-j,0}$, decrease. This reasoning can be seen in the right hand side of the FOC 1.27 for seller j and type θ: the lower is the public reputation of one's competitor, the more weight is put on the second term of the sum on the RHS of 1.27, which, given the decreasing differences property of the expected revenues function, is also the bigger term. In other words, when competitor's reputation $\mu_{-j,0}$ and respectively his success probability r_{-j} are low, seller j has higher incentives to exert higher effort to attain a success and maintain/improve his reputation.

The number of bidders impacts the future profits through two channels: on the one hand, in line with Bapna, Dellarocas, and Rice, 2010 analysis, a higher number of bidders reduces the effect of relative reputations on buyers' auction selection strategies, and on the other it leads to an increase in the expected price in any single auction due to a larger pool of bidders available. Expected higher future profits trigger the seller to employ higher effort in the present. When the number of bidders increases, their assortative bidding behavior converges towards perfectly randomizing between sellers. This effect, as argued in the previous section, is due to the fact that a high total number of auction participants reduces the chances of each of them to win any particular auction and reduces the expected benefit in the event of winning an auction. Hence, for high competition among bidders (high N), their optimal strategy prescribes them to perfectly randomize between sellers. The change in buyers' bidding behavior as we vary N has a significant impact on sellers' expected profits and hence on their optimal effort. As the number of bidders increases, the relative reputations ranking becomes irrelevant as both sellers will receive bids equally probable from all bidders. In addition to the vanishing assortative bidding, ahigher number of bidders in each auction c.p. increases the chances that the bidders have higher valuations which leads to higher expected profits.

1.5 Final Remarks

On-line markets, loan securitization (or more broadly, asset backed securities) markets or debt markets are characterized by the intrinsic asymmetry of information regarding seller (or borrower) trustworthiness, as well as the effort that an originator, a seller or a borrower put in order to monitor a loan, deliver a good or efficiently manage a project. In addition, agents in such markets are usually non-anonymous, which means that reputation building can potentially arise as a way to tackle the adverse selection or moral hazard problems.

We have shown that, contrary to the common intuition about the effect of (oligopolistic) competition in a general market setting, a seller′ incentives to build a reputation through effort in order to deliver and complete a transaction decrease when competition occurs within a 2nd price auction environment and when the reputation of the competitor increases. This is an important insight, not only from a theoretical point of view, but also from a more practical perspective, since it allow us to shed light on the usefulness of reputation measures as a way to solve the "lemons problem" in on-line markets.

Crucially, in on-line markets such as ebay the public information structure available to sellers and buyers is the same as the one available to us researchers (except, perhaps, for potential email correspondence between parties). This implies that predictions made by buyers about the behavior or characteristics of a particular seller necessarily use public information which we can observe. This, together with the dynamic game framework could allow us to study quantitatively two identification problems related to the reduced-form estimation of the impact of seller reputation measures on revenues, and that have been omitted by recent empirical papers. Biases in the estimation of this elasticity might arise when reputation building and strategic interactions are not controlled for, since both reputations can be correlated in equilibrium. In other words, the omission of strategic behavior can bias the estimate of the reputation elasticity of revenues. Nonetheless, our theoretical results also hint that when there is a reasonable number of buyers in the market, the consequences of omitting information about other sellers could be milder.

1.6 Figures

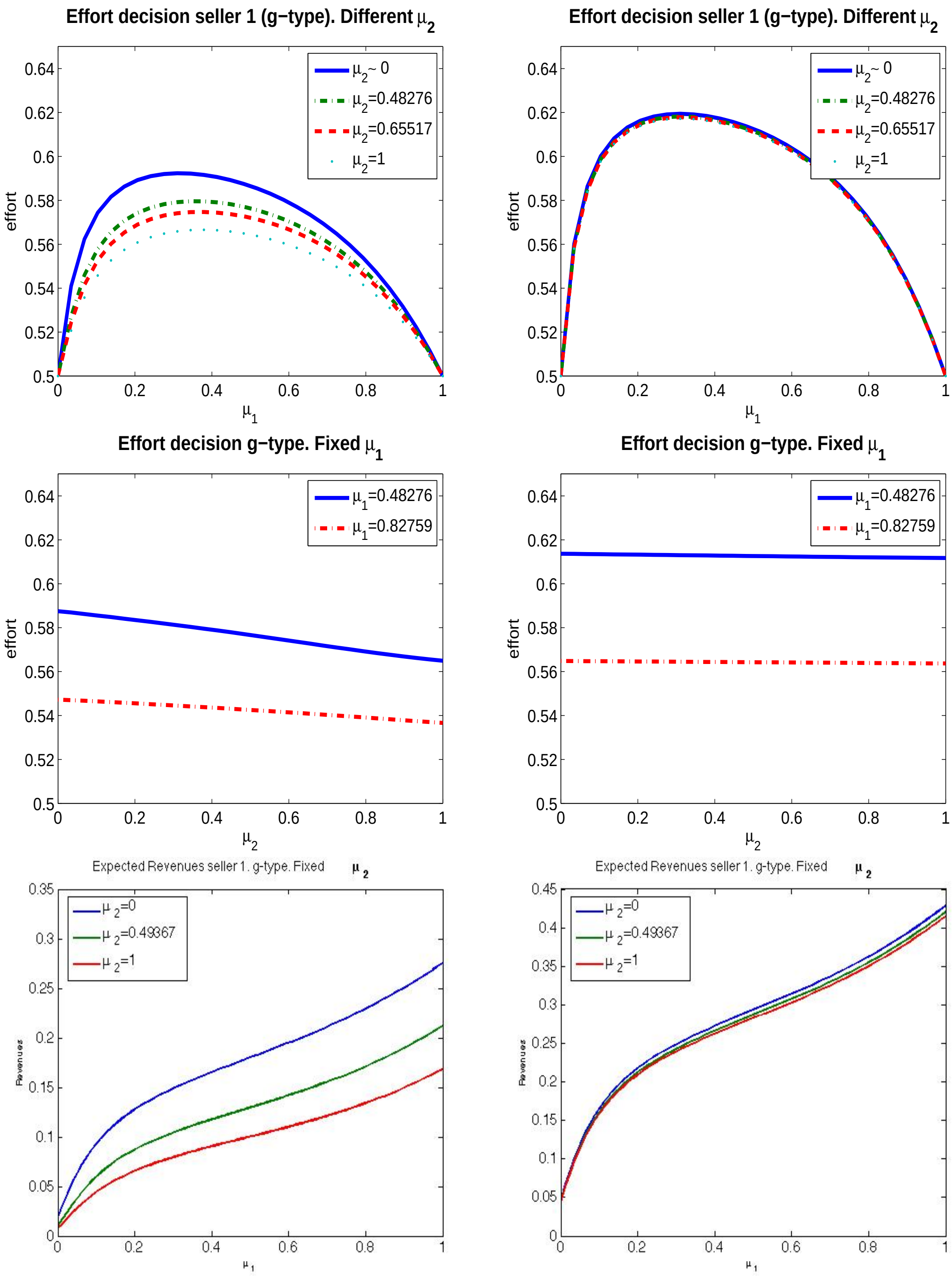


FIGURE 1.5: Equilibrium efforts in period 1 (top, middle) and PV profits (bottom). Left column: $N = 5$ Right column: $N = 50$

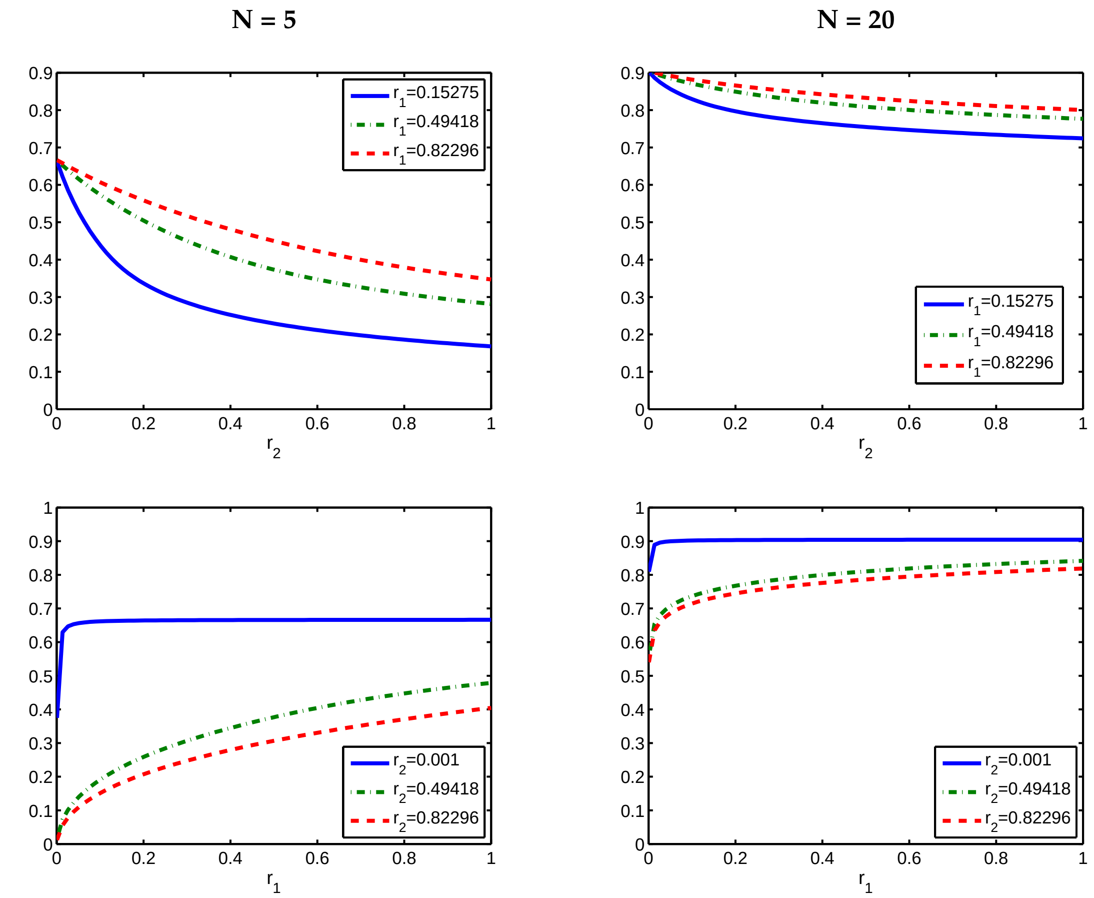


FIGURE 1.6: Expected second highest bid on **seller 1 auction**: fixed r_1 (top) and fixed r_2 (bottom). Left: $N = 5$ Right: $N = 20$

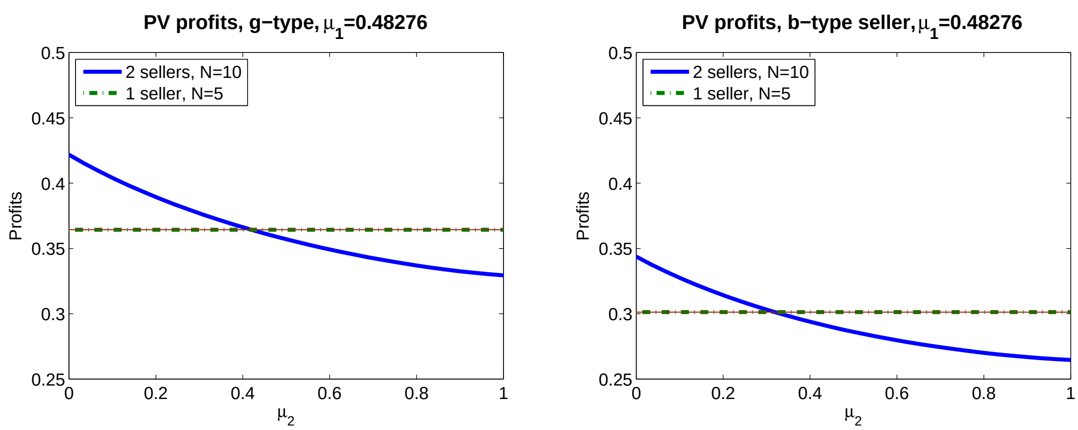


FIGURE 1.7: Present Value Profits (PV) for one seller, as a function of the competition's reputation, in the (i) one seller game, (ii) 2-sellers game. Left: "good" sellers (g-type). Right: "bad" sellers (b-type). For a proper comparison, the number of buyers in the 2-sellers case is 2x the number of buyers in the 1-seller case. In the figures: N=5 for the 1-seller case and N=10 for the 2 sellers case.

Chapter 2

Monetary Policy when Households have Debt: New Evidence on the Transmission Mechanism

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Keywords: household expenditure, micro data, housing tenure, monetary policy

2.1 Introduction

How monetary policy affects the real economy is one of the oldest and most intensively investigated topics in macroeconomics. In many of the models with nominal rigidities that are widely used in academic and policy circles, interest rate changes primarily affect the price of consumption today relative to the price of consumption tomorrow — the so-called intertemporal substitution channel. But the recent financial crisis has sparked renewed interest in whether other, less explored, mechanisms might complement, amplify or even dominate the quantitative effects implied by intertemporal substitution alone.

Two often discussed examples relate to the household balance sheet. In particular, the impact of interest rates on households, and the economy more generally, could be significantly affected by (i) the structure of the mortgage market, especially the prevalence of adjustable rate versus fixed rate contracts, (ii) the possibility that households

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with mortgage debt face some form of liquidity constraints. While a number of important recent theoretical and empirical contributions have improved our understanding of how the transmission mechanism may work, a detailed empirical analysis of which channels are quantitatively more important is still lacking. Furthermore, despite all the recent attention paid to the role of household indebtedness in shaping business cycle fluctuations, we are not aware of empirical studies that quantitatively examine the ways in which mortgage debt may affect the transmission of monetary policy. This paper attempts to fill the gap by providing evidence using household survey data for the United States and the United Kingdom.

In an important dimension, the U.S. and U.K. mortgage markets are often described as polar opposites. The majority of British products are characterized by relatively adjustable rates and shorter durations whilst most American products tend to have fixed rates and longer durations. If these structural differences in the mortgage markets play a significant role in the transmission of monetary policy we should observe two things. First, the direct interest rate effect on mortgage repayments should be significantly larger in the U.K. than in the U.S. We find evidence that this is indeed the case. Second, if this direct effect is the most quantitatively important channel, the difference between the expenditure of home-owners with a mortgage and home-owners without a mortgage should be significantly larger in the U.K. than in the U.S. In other words, the expenditure differences between these housing tenure groups should largely reflect the magnitude of the effect on mortgage repayments. However, our evidence does not support this second prediction.

On the other hand, if households with debt cannot easily smooth consumption (because, for instance, additional equity extraction is constrained by collateral values or by transaction costs associated with accessing illiquid wealth), the change in mortgagors' spending should be significantly larger than that of outright owners in both countries. Furthermore, in general equilibrium, there should be an effect on household income for all groups, implying a higher marginal propensity to consume for households with debt. If this mechanism is quantitatively more important than the direct cash flow effect, there should be similar expenditure differences between mortgagors and outright owners in two relatively different mortgage markets like the U.S. and U.K. Our evidence is indeed consistent with this hypothesis.

In looking at the disaggregated effects of monetary policy, we face a number of empirical and econometric challenges. Specifically, we need good quality micro data on expenditures and income, together with information on household balance sheets over a long period of time. Few, if any, datasets include this information. We therefore use rich micro data from the Living Costs and Food Survey (for the U.K.) and the Consumer Expenditure Survey (for the U.S.) and focus on a household's housing tenure status — specifically whether they own their home with or without a mortgage — as a proxy for their balance sheet position. Housing tenure status is then used to aggregate individual households into a group with debt (mortgagors) and a group without debt (outright home-owners). To tackle the reverse causality problem between interest rates and consumption, we measure monetary policy shocks using the identification strategy of Romer and Romer, 2004 for the United States and applied to the United Kingdom by Cloyne and Huertgen, 2015.

Our main empirical findings can be summarized as follows. Following a change in monetary policy, in both countries households with mortgage debt adjust their expenditure considerably more than outright owners without debt. The heterogeneity is far more pronounced for durable goods than for non-durable goods and services. The direct effect on mortgage repayments in dollars is nearly three times larger in the U.K. than in the U.S. (for a cumulative interest rate change of the same size). But, even for the U.K., the repayment effect in dollars is still much smaller (both statistically and economically) than the dollar change in total expenditure. The income of *all* housing tenure groups, however, responds significantly following a monetary policy shock, and these income changes are of the same order of magnitude as the mortgagors' expenditure response.

We interpret the similar and sizable response of expenditure relative to income for mortgagors in both countries as suggestive evidence that (i) households with mortgage debt tend to be liquidity constrained and (ii) the general equilibrium effect of monetary policy on income seems quantitatively more important than the direct effect of interest rate changes on cash-flows. We also show that, in both countries, between 40% and 50% of households with mortgage debt tend to live with net liquid wealth below half of their monthly income. This group therefore seems to feature a significant proportion of households who are liquidity constrained despite possessing sizable illiquid assets (i.e. they are 'wealthy' hand-to-mouth in the definition proposed by Kaplan and Violante, 2014).

To corroborate the interpretation of our empirical findings, we lay out a simple heterogeneous agent model featuring a liquidity constraint and multi-period debt (where we can vary a single parameter to reflect the different average fixed-rate contract durations in two mortgage markets like the U.S. and U.K.). We first employ an exogenous credit limit. This demonstrates the quantitative importance of a simple liquidity constraint in a world where two economies are subject to a different mix of adjustable rate and fixed rate contracts. We then extend the framework to include a constraint tied to the collateral value of a mortgagor's housing assets and an endogenous housing tenure decision between buying a property with a mortgage and renting. Both models are able to replicate qualitatively our empirical findings. The performance of the extended model, however, appears quantitatively superior, suggesting that additional amplification mechanisms — whether in the form of a (housing) collateral constraint as in Iacoviello, 2005 and Kiyotaki and Moore, 1997 or in the form of some transaction costs associated with accessing illiquid wealth as in Kaplan and Violante, 2014 — play an important role in fully accounting for the aggregate and heterogeneous effects of monetary policy on the real economy.

Related literature. This work relates to four strands of the literature. First, we provide empirical support for theoretical mechanisms that emphasize the role of debt-constrained agents in the transmission of economic shocks. Prominent examples include Kaplan and Violante, 2014, Kaplan, Moll, and Violante, 2015, Bayer et al., 2015, Ragot, 2014, Eggertsson and Krugman, 2012 and Iacoviello, 2005.

Second, we contribute to the large body of evidence on the relationship between the housing market, credit and real activity, with Mian, Rao, and Sufi, 2013, Mian et al., 2015, Guerrieri and Iacoviello, 2014, Aladangady, 2014, Jorda, Schularick, and Taylor, 2014, Justiniano, Primiceri, and Tambalotti, 2015; Justiniano, Primiceri, and Tambalotti, 2014 and Cloyne and Surico, 2015 being recent examples. While we share an emphasis on developments in the mortgage market, unlike most of these contributions we use expenditure survey data to explore interesting dimensions of heterogeneity which can be used to identify the groups that are most likely to drive the aggregate effects of monetary policy.

Third, we relate to a growing literature studying how the structure of the mortgage market (and repayments in particular) affects the transmission of monetary policy. On the theoretical side, this includes Calza, Monacelli, and Stracca, 2013 (who also explore this empirically using aggregate data), Garriga, Kydland, and Sustek, 2013 and Auclert, 2015a. More recent empirical work using micro data includes Keys et al., 2015 and Di Maggio, Kermani, and Ramcharan, 2015. While we also consider the response of mortgage payments, our empirical analysis focuses on the joint response of income, non-durable consumption and durable expenditure. We show that considering all these responses is important for assessing the relative merits of competing views of the monetary transmission.

Finally, our findings complement the evidence from an increasing number of studies, including Coibion et al., 2012, Gornemann et al., 2012, Sterk and Tenreyro, 2015 and Wong, 2015a, which report some heterogeneity in the consumption response to monetary policy across demographic groups. In contrast, we focus on household debt positions and find strong evidence of heterogeneity over and above any possible heterogeneity arising from demographic factors alone.

Structure of the paper. The rest of the paper is structured as follows. Section 2.2 presents the datasets, discusses the identification of the monetary policy shocks and the strategy of grouping households by their heterogeneous debt positions. The baseline estimates are reported in Section 2.3, together with evidence that a significant portion of mortgagors are 'wealthy' hand-to-mouth. We also show that the general equilibrium effect on income is quantitatively more important that the direct effect of interest rate changes on interest cash-flows. Further empirical results are presented in Section 2.4, where we assess whether other traits such as demographics or compositional changes in the housing tenure groups may be responsible for our findings. In Section

2.5, we examine the extent to which the predictions of a class of heterogeneous agent models with a liquidity constraint and multi-period debt are consistent with our novel empirical findings on the transmission of monetary policy. The Appendices provide some additional results, detail the derivation of the theoretical models and report the response of house prices and housing equity withdrawal to a monetary policy shock.

2.2 Data and empirical framework

In this section, we describe our main sources of household survey data and the variables we use. We then lay out the strategy to group individual observations into pseudo-cohorts using housing tenure status to proxy a household's debt position and discuss the identification of the monetary policy shocks. Finally, we present the empirical specification that we use.

2.2.1 Household survey data

In order to measure how different types of consumers respond to monetary policy across spending categories, we use household survey data with rich coverage of expenditure variables. For the U.K., this is the Living Costs and Food Survey (LCFS), previously known as the Family Expenditure Survey (FES). For the U.S., we use the Consumer Expenditure Survey (CEX).

We make use of detailed information on weekly expenditures both on non-durable goods and services, and on durable goods (excluding housing and rental-related costs), as well as on household income.[2] The latter is defined as labor income (wages and salaries) plus non-labor income (income from investments and social security payments), net of taxes paid by each household. In the appendix, we provide a more detailed description of the variable definitions and the sample restrictions.

In addition, the survey provides information on other two sets of variables which will be useful for our main empirical estimation: (i) demographics, in particular household size and the year of birth of the household head, (ii) mortgage payments for households with outstanding debt. On the one hand, the information on birth years will be used to verify that the heterogeneity we uncover across housing tenure groups does not reflect (omitted) differences in life-cycle positions (as opposed to genuine differences in debt positions that are independent of demographics). On the other hand, the mortgage payments data will be used to quantify the extent to which differences in mortgage market structures across the two countries, specifically that the share of

[2] As documented in the household consumption literature (Aguiar and Bils, 2015 and Attanasio, Battistin, and Leicester, 2006 using the U.S. Consumer Expenditure Survey (CEX) and Crossley, Low, and O'Dea, 2012 and Brewer and O'Dea, 2012 using the U.K. LCFS), the non-durable consumption and the durable expenditure reported by households fall short, when aggregated, of the figures in the U.K. and U.S. National Accounts. Following this literature, we adjust the household data in the following way: in each quarter and *for each* household, we scale-up the reported expenditure categories (either non-durable or durable) using the inverse of the ratio of aggregated expenditure (implied by the LCFS/CEX) to the values in the National Accounts.

adjustable rate products in the U.K. is significantly larger than in the U.S., may change the effectiveness of monetary policy. The information on household size will allow us to conduct the analysis at the per capita level.

We convert weekly data into a quarterly time series using the date of interview. The resulting series is then deflated by the Retail Prices Index (excluding mortgage interest payments) for the U.K. and the Consumer Price Index for the U.S. to convert the data into real series. Our sample covers 1975 to 2007 for the U.K. and 1981 to 2007 for the U.S.. The key variables of interest are available in the FES from the mid-1970s whereas the CEX begins in 1981. Our focus is on interest rate changes, so we deliberately stop just prior to the financial crisis, excluding the period of "unconventional" monetary policy.

2.2.2 Grouping households into pseudo-cohorts

The first empirical challenge we face is that, to our knowledge, there are few, if any, datasets that contain disaggregated information on *both* (i) wealth/household balance sheets and (ii) a rich array of expenditure categories at the household level over a sufficiently long period of time. Unfortunately, the LCFS and the CEX are no exceptions but they do record detailed expenditure and income data as well as information on housing tenure positions, namely whether a household lives in rented accommodation, is an owner-occupier with a mortgage or owns the property outright without a mortgage. As shown in Cloyne and Surico, 2015 for the U.K. — and further explored here for both the U.K. and the U.S. — housing tenure status is an effective proxy for the household debt position. This allows us to bypass the drawback that most surveys with rich expenditure data contain little other information about household balance sheets. Accordingly, and in keeping with the tradition of Browning, Deaton, and Irish, 1985, we employ a grouping estimator to aggregate individual observations into pseudo-cohorts by housing tenure.

At this point, it is worth discussing two potential concerns about grouping households by their housing tenure status. The first concern is about compositional change, namely that a household may change housing tenure status in response to a monetary policy shock. The second concern is about selection, namely that the assignments into the mortgagor and outright owner tenure groups are not random and some other (unobserved) characteristics may be potentially responsible for the heterogeneity in our estimated responses.

In terms of compositional changes, the time series of the tenure shares in Figure 2.1 are clearly slow-moving. The variation in monetary policy we exploit, however, occurs at a much higher frequency (as can be seen from Figure 2.2). In Section 2.4, we formally provide evidence that compositional change seems unlikely to be driving our results by showing that the monetary policy shocks do not significantly affect the shares of households in each housing tenure group.

Turning to the issue of selection, a number of factors seem to make it less severe in our context. First, the choice to group by housing tenure can be motivated from various theoretical frameworks. For example, the distinction between households with and without mortgage debt fits well with the predictions of a range of theoretical heterogeneous agent models that would imply heterogeneous expenditure responses to monetary policy shocks. Prominent examples include the financial accelerator mechanism in Iacoviello, 2005, following Kiyotaki and Moore, 1997, where a collateral constraint governs the household's ability to extract equity from housing and the 'wealthy' hand-to-mouth framework in Kaplan and Violante, 2014 where households have to pay a transaction cost to access their net illiquid wealth. Since our mortgagor group appears to have a close mapping with the constrained households in these models, our estimates can shed light on the empirical relevance of these theoretical mechanisms.

Second, we document in Section 2.3 that between 40% and 50% of mortgagors live with net liquid wealth below half of their monthly income and therefore appear far more likely to be liquidity constrained than outright owners. Hence, housing tenure seems a good predictor of the household's balance sheet position.[3]

Third, one of our main findings — namely that the direct effect of monetary policy on interest payments seems less important in the aggregate than the indirect effect — is not based on a comparison across housing tenures but on the relative magnitude of the responses of two different variables (mortgage payments and income) for the *same* household group (mortgagors). Furthermore, we show that income responds by a similar order of magnitude for all housing tenure groups.

Finally, in Section 2.4, we discuss other possible explanations and show that our results for mortgagors and outright owners are robust to controlling for these other mechanisms. In particular, we show that the expenditure response of young mortgagors appears to be similar to the response of middle-aged mortgagors. The responses of middle-aged outright owners and middle-aged mortgagors are, however, still significantly different. We interpret this further evidence as suggesting that the heterogeneity that we uncover based on housing tenure exists over and above any possible heterogeneity purely due to demographics.

Selected descriptive statistics for all housing tenure groups are reported in Appendix A. There appears to be some (small) differences across the distributions of per-capita income and across the shares of post-compulsory educational attainment between mortgagors and outright owners. While the age difference between these two groups seems larger, (i) the age distributions for mortgagors and outright owners still overlap significantly and (ii) as already noted, the heterogeneous responses across housing tenure groups are not over turned in the sensitivity analysis of Section 2.4 where we further control for demographics.

[3] Kaplan and Violante, 2014 for the U.S., Cloyne and Surico, 2015 for the U.K. and Kaplan, Violante, and Weidner, 2014 for a number of other advanced economies show that households with mortgage debt also tend to have higher levels of unsecured debt relative to outright owners.

Before ending this section, it is also worth noting that we have information on the remaining group of households in the sample, namely renters. These are a relatively heterogeneous group comprised of social renters (those renting from local authorities and housing associations or benefiting from certain government transfers) and private renters (who — on average — tend to represent around 10 % of the population). The balance sheet information presented in Cloyne and Surico, 2015 suggests that renters are likely to be an interesting proxy for the type of liquidity constrained households typically found in one asset models. While our main focus in Section 2.3 is on the comparison between home-owners with mortgage debt and home-owners without mortgage debt, the results for renters are of independent interest as one may expect their responses to be more similar to those of mortgagors than outright owners. The findings for renters' expenditure and income responses are explored in Section 2.4.

2.2.3 Identification

Our goal is to examine the effect of monetary policy on the spending and income of different *groups* of households. As such, we face the usual macroeconomic reverse causation problem: the economy responds to movements in monetary policy, but monetary policy also responds to developments in the macroeconomy. To identify unanticipated changes in the short-term interest rate we need a monetary policy shock series that can be used for estimation.

There is a vast literature on the identification of monetary policy changes. Older approaches, mainly developed for the United States, relied on timing restrictions and a Choleski decomposition of the variance-covariance matrix of the residuals from a Vector Autoregression, such as Christiano, Eichenbaum, and Evans, 1996; Christiano, Eichenbaum, and Evans, 1999. But when applied to the United Kingdom, this method produces a large rise in inflation following a monetary contraction, the so-called price puzzle, even after controlling for variables shown to ameliorate this issue for the U.S. (Cloyne and Huertgen, 2015). Another popular approach for the U.S. was introduced by Romer and Romer, 2004. This method first constructs a measure of the target policy rate (since the effective Federal Funds Rate is moved around by other factors than just policy decisions) and then regresses the change in the target rate around the policy decision on a proxy for the information set available to the policymaker just prior to that decision. This information set includes a range of real time indicators and forecasts to reflect the forward-looking nature of monetary policy. Cloyne and Huertgen, 2015 construct a measure for the U.K. employing this methodology and show that it improves on conventional VAR methods. Hence, we use an updated version of the Romer and Romer, 2004 shock series for the U.S. (whose original analysis ended in 1996) and the Cloyne and Huertgen, 2015 shock series for the UK.[4] One particular advantage of using

[4]Unfortunately, the length of the sample that we consider prevents us from using a high frequency identification strategy as in Gertler and Karadi, 2015.

shocks based on the Romer and Romer, 2004 method is that we have two comparable series across the two countries we study.

The shock series match the micro-data sample periods, which are from 1975 to 2007 for the U.K. and 1981 to 2007 for the U.S. The shock series deliberately stop before the recent financial crisis, when the policy rate hit the zero bound in both countries. To boost the number of household observations used to generate the pseudo-cohorts at each point in time, we aggregate household survey variables to a quarterly frequency. The original shock series are monthly but, following Romer and Romer, 2004 and Coibion, 2012, we sum up the monthly innovations to obtain a quarterly series. The construction of the U.K. series also allows for a break in 1993 around the adoption of the inflation targeting framework. The monetary policy shock series for the two countries are shown in Figure 2.2.

Cohort-specific Granger causality. The shock series we use should already be regarded as monetary innovations from a macroeconomic perspective. But there is still a concern that the monetary policymakers might have been reacting to the conditions in particular groups. While some of this should be captured in the policymakers' forecasts, for example if they were concerned about developments in the housing market, it is useful for our purposes to test whether the U.S. and U.K. shock series can be predicted by movements in cohort level consumption and income. Finding that these shocks are unpredictable on the basis of cohort level concerns would therefore be reassuring. Specifically, we conduct Granger causality tests based on a VAR which contains consumption, expenditure and income per capita for each household group. We cannot reject the hypothesis that the cohort-specific time-series from household survey data (as well as the aggregate time-series from national statistics) do not Granger cause the monetary policy shocks in each country.

2.2.4 Empirical specification

Using the two monetary policy shock series, our empirical specification closely resembles Romer and Romer, 2004. Accordingly, we regress the variable of interest on a distributed lag of the monetary policy shocks. As in Romer and Romer, 2004, we also control for the lagged endogenous variable as is common in exercises with relatively short samples. Specifically, we estimate the following equation:

$$X_{i,t} = \alpha_0^i + \alpha_1^i trend + B^i(L)X_{i,t-1} + C^i(L)S_{t-1} + D^i(L)Z_{i,t-1} + u_{i,t} \qquad (2.1)$$

where $X_{i,t}$ is real non-durable consumption, durable expenditure or income recorded by households interviewed at time t.[5] The monetary policy shocks are denoted by S and Z is a vector with additional controls, including quarterly dummies. The α terms

[5]Households interviewed at time t are typically asked to report expenditure over the previous three months (with the exception of non-durable consumption in the LCFS which refers to the previous two weeks). To eliminate

represent intercepts and coefficients on a time trend polynomial, with a break in 1993 for the U.K. and no break for the U.S.. Finally, $i \in Mortgagors, OutrightOwners, Renters$ represents the housing tenure group each household belongs to. The orders of the lag polynomials are chosen using an optimal lag length criteria, namely the corrected AIC.[6] Standard errors are bootstrapped using a recursive wild bootstrap.

2.3 Main findings

In this section, we present the results from estimating our benchmark specification (3.1) with both aggregate and cohort level data. In order to make the results comparable with the previous literature, all the impulse response functions (IRFs) are computed by simulating a 25 basis points (bp) *temporary* cut in the policy rate. All figures display point estimates together with bootstrapped 90% confidence bands generated from 5000 resamples. We begin with evidence using official national statistics before presenting results for non-durable consumption, durable expenditure, income and mortgage repayments using household survey data. In the last part of this section, we transform the percent changes estimated from our IRFs into the implied dollar changes and find that the mortgagor group's expenditure responds the most relative to income. Finally, we use data from the American Survey of Consumer Finance and the British Household Panel Survey to provide independent evidence on the extent to which mortgagors may be liquidity constrained despite owing sizable illiquid assets.

2.3.1 Evidence from official national statistics

Before exploring the response of different household groups, it is useful to examine the aggregate response of non-durable expenditure, durable expenditure and household income from the U.K. and U.S. official aggregate statistics. These results are presented in Figure 2.3. We find that a cut in the policy rate raises durable expenditure, non-durable expenditure and disposable income. More specifically, a 25 basis point monetary policy expansion leads to (i) a persistent (but small) rise in non-durable consumption, which peaks at around 0.2% after about 10 quarters for the U.K. and 0.1% after 11 quarters for the U.S., (ii) a larger *percentage* increase in durable expenditure (peaking at 1.2% for the U.K. and 1% for the U.S.), consistent with the evidence in Barsky, House, and Kimball, 2007 and Sterk and Tenreyro, 2015 and (iii) a rise in household income (that reaches its maximum at 0.4% in the U.K. and just below 0.3% in the U.S.). While the U.K. aggregate variables tend to exhibit a slightly larger adjustment, the differences between the two countries are not large or significant. But to establish the possible role

some of the noise inherent in survey data, $X_{i,t}$ is smoothed with a backward-looking (current and previous three quarters) moving average.

[6] We have also explored a generalized specification where X is a vector, but with similar results. In addition, we have experimented with including the contemporaneous value of the shock and with assuming a different type of trend. In all cases, our results are robust.

of indebted households, including the direct effect of interest rate changes on mortgage payments, we need to look at the more disaggregated behavior of different housing tenure groups.

2.3.2 Evidence from household expenditure survey data

We now explore the heterogeneous response by housing tenure status. The results comparing mortgagors and outright owners can be seen in Figure 2.4 for the consumption of non-durable goods and services, in Figure 2.5 for the durable good expenditure, in Figure 2.6 for mortgage payments (mortgagors only) and in Figure 2.7 for income. The corresponding variables for renters are discussed in Section 2.4 and presented in Appendix Figure B.10. In each chart, the top row refers to the U.K. and the bottom row to the U.S.. The left column corresponds to the behavior of mortgagors, the right column refers to outright owners.

Beginning with the response of non-durable consumption in Figure 2.4, the response of mortgagors tends to be larger than the adjustment made by outright owners. Specifically, the response of households with mortgage debt in the U.K. peaks at 0.3% after about 10 quarters but the response of households without debt is never statistically different from zero, suggesting that the behavior of mortgagors drives the aggregate result for non-durables in Figure 2.3. For the U.S. the pattern is similar, with the peak response in the left column reaching about 0.25%.

Strong evidence of a heterogeneous effect between groups can also be seen in the response of durable expenditures in Figure 2.5. The heterogeneity between housing tenure groups is now starker. The response of U.K. mortgagors' expenditure peaks at around 1.2%, whereas the reaction of outright owners' durables is statistically indistinguishable from zero. The bottom row paints a similar picture for the U.S., with the significant and persistent response of households with debt peaking around 1.2%, and driving the aggregate durable response reported in Figure 2.3.

In summary, in response to a monetary policy shock, the percentage changes in expenditure tend to be large and significant for mortgagors but small and insignificant for outright owners. Furthermore, this heterogeneity is far more pronounced for durable goods. Importantly, there does not seem to be any significant difference across the two countries. While the lack of heterogeneity in the response of mortgagors between the U.K and U.S. may already suggest a limited role for the different mortgage market structures, in the next subsection we tackle this directly.

2.3.3 The response of mortgage payments and income

A possible explanation for the sensitivity of mortgagors' expenditure to monetary policy is that a change in the interest rate triggers a *direct* effect on mortgage payments. More specifically, a monetary expansion should lead to a fall in interest payments for borrowers and a fall in interest income for savers. But for this redistribution to have an impact on the wider economy, it would still need to be the case that borrowers (the mortgagor group in our context) are credit constrained, so that these cash flow effects do not net-out in the aggregate. This is where the comparison between the U.S. and the U.K. is an ideal assessment of this hypothesis. The U.S. tends to have longer duration mortgage contracts and a dominance of fixed rate products whereas the U.K. has shorter duration contracts and more variable rate products. As such, it should be that the effective lending rates facing indebted American households are relatively insensitive, leading to a smaller change in their repayments following a monetary expansion.

As discussed in Section 2.2, we use household mortgage payments from the LCFS and the CEX. Figure 2.6 reports the impulse response function for the *percentage change* in this variable following a cut in the policy rate, revealing that mortgage payments fall significantly following a monetary expansion. The response for the U.K. appears considerably faster than in the U.S., peaking at around 0.7% versus the U.S. peak of around 0.4%. As we show in the next section, these estimates imply that the direct effect of interest rate changes on repayments is nearly three times larger in the United Kingdom than in the United States. This is intuitive given the differences in the mortgage market structures. Our results therefore suggest that the distinction between adjustable and fixed rates may play some quantitative role in the transmission of monetary policy.

A different (but not mutually exclusive) explanation for the mortgagors' expenditure response, however, is that the bulk of the effects of the interest rate change is in fact *indirect*. This can be best understood by looking at joint responses of consumption *and* income. Figure 2.7 shows that income responds significantly for both housing tenure groups and in a similar proportion across the two countries, with the average point estimates for mortgagors and outright owners typically around $0.3 - 0.4\%$.[7] In the next section, we show that the percentage changes in Figure 2.7 map into dollar changes in income that are significantly larger than the dollar changes in mortgage payments induced by the monetary policy shock.

2.3.4 Inspecting the mechanism

The evidence above is consistent with the presence of both direct and indirect effects of interest rate changes. But this does not yet establish whether one of the channels is quantitatively more important in accounting for the absolute magnitude of the response of mortgagors' expenditure. This requires converting the estimated percent changes in the IRFs into dollar changes and then assessing how the dollar change implied by the cash flow effect (the direct move in mortgage payments) and the general equilibrium effects (captured by the response of income) compares with the dollar change in total expenditure.

[7] Trimming the top 5% of the financial income distribution does not overturn our main findings but reduces the uncertainty around the point estimates of the income responses by about 25%.

To this end, we convert the percentage changes in the impulse response functions of Figures 2.4 to 2.7 into an equivalent dollar change using the average value of each variable for each cohort in each country (as reported in the Appendix Table B.1). We use the U.S. price level in 2007 and the average exchange rate between pounds sterling and U.S. dollars in that year to compute the cumulative change over the period of the simulated impulse response functions.[8]

In Table 2.1, we report the results (with 90% bootstrapped confidence bands in square brackets). The first three columns correspond to non-durable consumption, durable expenditure and mortgage payments. The last column reports the cumulative dollar change in household income. Panel A refers to the United Kingdom and Panel B to the United States. Within each panel the first row corresponds to mortgagors, the second row refers to outright owners without mortgage and, for completeness, the third row is for renters. In interpreting these magnitudes, it is worth noting that, while the absolute numbers in this table may appear small, we are considering a small and temporary change in interest rates (25 basis points on impact and then returning to zero relatively quickly over the forecast period).[9] These numbers are also an average of the effect on loan rates for newly originated mortgages and the effect on existing (adjustable and fixed rate) loans.

A number of important findings emerge from Table 2.1. First, the dollar change in mortgage payments in the U.K. is nearly three times larger than in the U.S., consistent with the notion that the share of adjustable rate products has been historically significantly higher in the U.K. (Besley, Meads, and Surico, 2013). Second, the change in average mortgage payments is significantly smaller than the overall change in mortgagors' spending on non-durable and durable goods. This suggests that the direct effect of the interest rate change on cash-flows alone does not generate sufficient resources to fund the expenditure change that we observe in the data.[10] Third, in contrast, the dollar change in mortgagors' income is of a similar order of magnitude as the dollar change in mortgagors' expenditure. But it is also not statistically different from the dollar change in income for the other tenure groups. The response of income for all groups seems most likely to reflect the general equilibrium effects of monetary policy on the macroeconomy. Fourth, the dollar changes in expenditure for outright home-owners

[8]The response of the policy rate to the monetary policy shock tends to be slightly more persistent in the U.S. than in the U.K., as can be seen in Coibion, 2012 and Cloyne and Huertgen, 2015. To make the magnitudes comparable, we rescale the U.K. numbers by the ratio of the cumulated response of the U.S. Federal Funds Rate and the cumulated response of the U.K. Bank Rate. This is like rescaling by the relative movement in the long-rate.

[9]Note that the size of our monetary policy shock is about twelve times smaller and at least five to six times less persistent than the shock in Keys et al., 2015 and Di Maggio, Kermani, and Ramcharan, 2015.

[10]The size of the change in the average mortgage payments in Table 1 refers to a temporary 25 basis points cut in the policy rate and accords well with a back-of-the-envelope calculation using an *effective* mortgage duration of ten years, an *effective* loan to value ratio on outstanding debt of 0.5 and, for the U.K., the average house value from the Land Registry since 1995 (and from Halifax before then) as well as a share of mortgages on adjustable rates of 45%. This yields an average change in U.K. mortgage payments of 168 US dollars. Replacing the UK share of mortgage contracts on adjustable rates with a share of 15% for the U.S., we obtain a value of 56 dollars.

are never statistically different from zero, despite significant movements of their income in both countries. It is therefore the mortgagor group who adjusts expenditure significantly relative to income, which is consistent with the notion that households with debt are characterized by higher marginal propensities to consume.

Wealthy hand-to-mouth mortgagors. To corroborate the conclusion of Table 2.1, we draw on independent evidence on the extent to which households with mortgage debt may face a liquidity constraint, using the British Household Panel Survey (BHPS) and the American Survey of Consumer Finance (SCF) for the (multi-year) waves that correspond to our baseline samples.[11] As these households own sizable illiquid assets (in the form of housing) and respond significantly to changes in interest rates, they appear to fit well the definition of 'wealthy' hand-to-mouth (WHTM) put forward by Kaplan and Violante, 2014. More specifically, we define a household as 'wealthy' hand-to-mouth if at any given point in time both (i) their net illiquid wealth is positive and (ii) their net liquid wealth is less than half of their total monthly household labor income.[12]

In Figure 2.8, we report the share of mortgagors who are 'wealthy' hand-to-mouth for pairs of temporally close waves in the BHPS and SCF. While there are more waves than displayed for the SCF, there are only three waves of the BHPS over our sample period (but we have confirmed that similar results emerge from the SCF waves that we have not reported). The clear message from this chart is that between 40% and 50% of households with mortgage debt have very low levels of liquid wealth, suggesting that they may find themselves liquidity constrained. In Figure B.6 of Appendix C, we also show that most WHTM agents in the sample do have a mortgage. This section has therefore presented further evidence to support the idea that many mortgagors may behave in a liquidity-constrained manner, with important implications for the aggregate effects of monetary policy.

[11]These surveys do not contain wealth information at a sufficiently high frequency to be used for our main analysis and they lack rich consumption data over a long period of time.

[12]When constructing the relevant household income and wealth measures, we select variables to make the concepts of net liquid and illiquid wealth as consistent as possible across the two datasets. The BHPS only reports quantities for overall investment and debts whereas for specific assets it only records whether these are held or not. The SCF, in contrast, does provide quantities for particular assets and overall investment. Furthermore, the assets on which information is provided differ slightly between surveys. Accordingly, net liquid wealth in the U.K. is constructed as total amount of liquid savings and investments (National Savings Bank Accounts and Cash ISAs or TESSAs, Premium Bonds, Stocks and shares ISAs or PEPs) minus non-mortgage debt (Hire purchase agreements, Personal Loans, Credit and store cards, DWP Social Fund loans). Following Kaplan and Violante, (2014), net liquid wealth in the U.S. is the value of checking, saving and MM accounts, directly held mutual funds, stocks, bonds and t-bills, net of outstanding unsecured debt. Net illiquid wealth in the U.K. is measured using a binary variable which takes value 1 if housing equity> 0 or the household has positive investments in (relatively) illiquid instruments such as National Saving Certificates, NS/BS insurance bonds, private pensions, non-regular savings. Net illiquid wealth in the U.S. is the value of housing equity (housing value - mortgage debt) plus pension/retirement funds, life insurance, saving bonds and certificate of deposits.

2.4 Further results

In the previous section, we showed that mortgagors tend to alter their overall expenditure far more than outright owners following a change in monetary policy. We also provided evidence on the relative response of expenditure and income, suggesting that mortgagors behave in a manner consistent with them facing liquidity constraints. Finally, we showed that, in the data, about half of the mortgage group have low liquid wealth and that the majority of 'wealthy' hand-to-mouth households hold a mortgage. But one may still be concerned that our housing tenure distinction is simply picking up another (deeper) characteristic or changes in group composition over time. In this section we explore this issue further. We also consider the results for renters.

2.4.1 Demographics

An important issue is whether our housing tenure distinction is simply picking up life-cycle effects. To explore this issue, we follow the micro-econometric literature and focus on birth cohorts. We regard households as 'older' if the head was born before 1935, as 'middle-aged' if the head was born in the interval [1935, 1949] and as 'younger' if the head was born after 1949.

In Figure B.1 of the Appendix, we show the breakdown of our tenure groups by birth cohort. As expected, there is a prevalence of mortgagors among younger households and a prevalence of outright owners without mortgage debt among older households but, importantly, not all younger households are mortgagors and not all older households are outright owners. Furthermore, the middle-aged group is populated by even shares of all housing tenure cohorts.

Within each housing tenure group, we further sub-divide households into birth cohorts. We then consider three experiments to explore whether age/life-cycle considerations could be driving our results rather than mortgage debt per se. First, we investigate whether younger mortgagors respond more than older mortgagors. Second, we focus on whether the response of middle-aged mortgagors is similar to the response of middle-aged outright owners.[13] Third, we verify whether excluding households with a retired head makes a difference to our results. If the answers to these questions are all negative, we can be more confident that the heterogeneity across housing tenure groups documented in the previous section is *not* picking up omitted demographic factors.

The comparison between the first and the second columns in Appendix Figures B.2 and B.3 shows that younger and middle-aged mortgagors respond similarly. This is true both in terms of the magnitude and in terms of the significance of the point estimates (both for non-durable consumption and durable expenditure). This suggests that conditional on the housing tenure group, age — as proxied by birth cohort —

[13]Unfortunately, there are neither enough mortgagors in the older birth cohort nor enough outright owners in the younger birth cohort for us to look at these two other sub-groups.

does not seem to play a significant role in the transmission of monetary policy in any of the two countries. On the other hand, the comparison between the second and third columns in Appendix Figures B.2 and B.3 reveal that the changes in middle-aged mortgagors' expenditures are typically large and significant whereas the changes in middle-aged home-owners' expenditures are typically small and statistically indistinguishable from zero. We interpret these findings as further suggestive evidence that, conditional on the age/life-cycle position, household debt plays an important role in the monetary transmission mechanism.

Finally, we consider restricted samples from the LCFS and the CEX where we exclude households with a retired head. The results from this exercise are reported in Appendix Figures 14 and 15. The figures show that our baseline estimates in Section 3 are confirmed when imposing this restriction.

In summary, the findings from the previous section are not overturned when considering the impact of demographics. In particular, the heterogeneous responses associated with housing tenures status appear to hold over and above any possible heterogeneity associated with age or birth cohort.

2.4.2 Compositional changes

To interpret our estimates as the causal effect of monetary policy on the expenditure and income of mortgagors, we need that the policy change does not cause households to move from one housing tenure status to another. Note that this is likely to be more problematic, if anything, for the U.K. survey data which consist of repeated cross-sections, than for the U.S. survey data where, given the short panel dimension, we already consider only those households who have not changed housing tenure status between interviews.

Housing tenure shares. In this exercise, we examine whether the monetary policy shock triggers any net inflows or outflows into each of the house tenure groups. Specifically, we look at the response of the group shares. As can be seen in Figure 2.1, the very gradual rate at which home ownership has changed in both countries relative to the high frequency movements in the monetary policy series, already suggests a limited response of the tenure shares. In Figure B.7, we examine this formally. Each panel reports the response of the group shares for mortgagors, outright owners and renters. It is clear that none of the shares responds significantly, indicating that changes in monetary policy do not seem to trigger significant endogenous changes in the housing tenure status.[14] This is possibly unsurprising given that the shock is only 25 basis points and the dollar changes in income from Table 2.1 are not especially large in an absolute sense.

[14] While it may be theoretically possible that the inflows into one group might be offset by its outflows, it would seem difficult to think that *at the same time*, for example, some renters become mortgagors and other households with debt become renters following a monetary policy shock.

A propensity score approach. One way of addressing concerns about endogenous changes in group composition more explicitly is to apply the Attanasio, Banks, and Tanner, 2002 propensity score approach. Rather than grouping based on actual housing tenure, this approach groups households based on the probability of being a mortgagor, using exogenous household characteristics as predictors. Specifically, we run a probit regression over the full sample to generate individual predicted probabilities of having a mortgage based on a high order polynomial in age, education, a time trend and their interactions.[15] For households observed in quarter t, we compute the probability that they had a mortgage in the previous quarter. For these two periods, we classify households as 'likely' or 'unlikely mortgagors' if the probability in the first of the two periods is larger or smaller than the share of mortgagors in the sample.[16] We then take the growth in consumption across these two quarters for each group, from which we can construct the implied consumption series for each of the groups that we then use for estimation.

The results of this exercise are shown in Appendix Figures B.8 and B.9. As can be seen, the main findings of our earlier analysis are not overturned. The likely mortgagor group still exhibits stronger expenditure responses than the unlikely mortgagors, despite income increasing for both groups. The possibility of changes in group composition therefore does not seem to be driving our estimates.

2.4.3 Renters

Turning to renters, Figure B.10 in the Appendix shows that, for both countries, the percentage changes in non-durable consumption are similar in magnitude and significance to those of mortgagors. The responses of durable goods are, however, typically smaller (especially in the U.K.) and less precisely estimated than for the other cohorts, possibly reflecting the heterogeneous composition of the renter group. On the other hand, income moves in a similar manner to the other cohorts and by a significant amount. In both countries, the impulse response functions for renters translate into positive dollar changes in expenditure (Table 2.1). In particular, Table 2.1 shows that, in both countries, the response of non-durable expenditure for renters is greater than the response of durables. The difference between the two expenditure responses is also larger for renters than for mortgagors. The overall dollar change in expenditure relative to income appears far larger for renters than outright owners and is line with, or smaller than, the magnitude for mortgagors.

[15] To maximize the number of households in each quarter and cohort, we place no restrictions on the birth year of the household head in this exercise but include age among the demographic variables in the probit regressions. To sharpen the comparison between mortgagors and outright owners, we do not include renters in the analysis in this section. While these restrictions do not affect the point estimates from the propensity score method significantly, they improve their accuracy.

[16] As time variation in the probability of being a mortgagor may induce changes in the group composition, we use a constant threshold for determining the group of 'likely' mortgagors.

2.5 Theoretical insights

Our empirical results suggest four main findings (i) the expenditure response of mortgagors is far larger than the expenditure response of outright owners, (ii) the heterogeneity is more pronounced for durable goods, (iii) income responds significantly for all groups and (iv) the direct effect of interest rate changes in both countries is modest relative to the effect on expenditure and income.

To explore the possible mechanisms behind these results further, we need a framework that allows us to quantify the relative contribution of possibly competing channels. Crucially, given the structural differences between the U.K. and U.S. mortgage markets, the model should allow for a meaningfully distinction between fixed rate contracts with different durations (or a different mix of fixed and adjustable rate products). At the same time, we want the model to be tractable enough so that we can analyze the transmission of interest rate changes in general equilibrium. With the above goals in mind, we consider a simple environment with households that are heterogeneous in their degree of impatience. Households also derive utility from leisure, non-durable consumption and the flow of services associated with the stock of durable goods and the stock of housing that they choose to hold. In the full version of the model, we also allow for a choice between renting and owning. In addition, households can smooth consumption using a *long term* (multi-period) debt instrument.

The liquidity constraint. We assume that households are subject to a constraint in the amount they can borrow. We first present a simple version where the credit limit is fixed exogenously. This will already allow us to understand the differential response of constrained versus unconstrained households under fixed rate contracts with different maturities. Building on this, we then consider a richer version where borrowing needs to be backed by a collateralizable asset, in this case housing stock. In addition, households make a housing tenure decision between owning (possibly with a mortgage) and renting. While the exogenous credit limit version produces results qualitatively in line with the estimates in Section 2.3, we show that the presence of a collateral constraint provides one (but by no means the only) mechanism that can amplify the effects associated with an otherwise exogenous credit limit.

Multi-period debt. We assume that borrowing and lending can only go through a nominal, long-term, risk free bond (or mortgage). One unit of debt issued at t pays, starting in $t+1$, the sequence of nominal installments $1, \rho, \rho^2, \ldots$, which decay at a rate ρ, following Woodford, (2001). Different values of ρ conveniently map into different effective average durations of fixed-rate periods. For example, $\rho = 0$ would correspond to a case in which the entire stock of debt in the economy is renegotiated in every quarter. On the other hand, a value of $\rho = 0.8$ (0.974) translates into a fixed-rate duration of about one (seven) year(s), which we take as a rough approximation of the average effective duration in the U.K. (U.S.) mortgage market.[17]

Rest of the economy. The other elements of the model are relatively standard features of a New Keynesian environment, such as sticky prices, habit formation and a Taylor rule for setting the short-term nominal interest rate. Houses are in fixed supply and wages are flexible. The model includes durable expenditures following Mertens and Ravn, 2011 and Monacelli, 2009. Finally, it is worth emphasizing that our model, while sufficiently rich to look at all the variables in our empirical analysis, has been kept deliberately stylized to highlight the relative importance of the key theoretical mechanisms that are most likely to be at play in the data. Accordingly, our theoretical analysis abstracts from a number of features which — in the empirical literature on DGSE models — have been shown to be quantitatively important to generate the persistence in the response of the endogenous variables observed in the data. Details and derivations of the two models below, including the description of the production side, pricing and monetary policy, are confined to the Appendix.

2.5.1 An exogenous credit limit

We first consider the case where households face an exogenous and constant constraint. Specifically, suppose there is a fixed credit limit, $\Omega_t = \bar{\Omega} \ \ \forall t$ such that

$$S_t b_{t+1} \leq \bar{\Omega} \tag{2.2}$$

where b_{t+1} is the real value of the debt repayment at the end of period t and $S_t b_{t+1}$ is the current market value of outstanding debt. Note that for $\rho = 0$, b_{t+1} becomes the real value of the entire outstanding debt and S_t becomes the inverse of the one-period bond return. This set up is similar to the one analyzed in Eggertsson and Krugman, (2012), with the distinction that here we consider debt with maturities longer than one quarter.

Our objectives are twofold: (i) to see whether this simple framework can rationalize our empirical findings; (ii) to disentangle the importance of the direct and indirect channels. Specifically, the former is a *cash-flow* effect, through which the budget constraint of debtors is directly affected by the interest rate change. The latter is a wider,

[17] On average over our sample period, nearly half of products in the U.K. mortgage market were originated at a fixed rate, with the most popular deal being over two years (Besley, Meads, and Surico, 2013). Accordingly, we regard an economy with $\rho = 0.8$ (or one year fixed rate period) as being sufficiently close to the average effective duration in the U.K. mortgage market. As for the U.S., Doepke and Schneider, (2006) report that since the 2000s, the average duration of financial assets has been around four years. Given that mortgages are typically characterized by higher duration than any other asset, this would seem to represent a lower bound for our calibration. On the other hand, the pace of entries and exits in the mortgage market suggests that an average *effective* duration above ten years would probably be implausible. For the sake of concreteness, we set $\rho = 0.974$ (a seven year fixed rate duration) for the U.S. but we have verified that the set of impulse responses below are not sensitive to choosing any value between four and twenty years.

indirect, *general equilibrium* effect where other variables such as household income can move, possibly significantly, following a monetary policy shock.

To quantify the relative importance of the cash-flow channel, we solve and simulate the model under different values of the duration parameter ρ, with zero representing the extreme case of an economy which refinances its entire stock debt in every period. The values of ρ are calibrated to match the average effective duration in the U.K. and U.S. mortgage markets discussed above.[18]

In Figure 2.9, we report the effects of a 25 basis point cut in the policy rate on non-durable consumption (top row), durable expenditure (middle row) and income (bottom row). Two important results are worth noting. First, this simple set-up can replicate the two main dimensions of heterogeneity we uncover in the micro data: (i) for a given expenditure category, constrained agents respond more than unconstrained agents, (ii) for a given household group, the change in durable goods is larger than the change in non-durables. This reflects that fact that it is the stock of durables that enters the utility function of the households. As income also moves in the data for both groups, the expenditure responses in the model reinforce our interpretation of debtors as hand-to-mouth agents.

Second, the impulse responses under different mortgage market structures show that the effects of monetary policy *are* larger when ρ is smaller, consistent with both our empirical findings for mortgage repayments and the theoretical results in Auclert, 2015a. The reason why this happens in the model is simple: when an indebted (constrained) household is financing a given consumption stream through debt with short maturities, the proportion of the total amount that needs to be re-financed each period is higher. An unexpected decrease in the refinancing costs generates an improvement in household resources. But, importantly, this does not appear to be main driver of the large response of debtors' consumption, even in the empirically unrealistic case in which everyone refinances the entire stock of debt in every period (i.e. $\rho = 0$). The simple model therefore already seems to capture the main aspects of our empirical findings, though the absolute magnitudes of the responses tend to be smaller than the peak effects that we have estimated in the micro data.

2.5.2 An endogenous collateral limit

In this section, we relax the assumption of a fixed borrowing limit and assess the quantitative performance of an extended model which adds two features: (i) an endogenous collateral constraint, (ii) a housing tenure decision, choosing between how much housing to own and how much housing to rent, with the relative price determined endogenously.

[18] Importantly, and as shown in the appendix, the real interest rate and consumption/income in steady state are *independent* of the value of ρ. This ensures a meaningful comparison of impulse responses under economies with different values of ρ.

We embed the collateral constraint as in Iacoviello, (2005) into our multi-period debt framework, with mortgagors who can borrow up to a given fraction ϕ of their expected housing wealth next period:

$$S_t b_{t+1} \leq \phi E_t \left(\frac{q^h_{t+1} h_t \pi_{t+1}}{R_t} \right) \tag{2.3}$$

where q^h_{t+1} is the real house price, h_t is the housing stock, π_{t+1} is the inflation rate between t and $t+1$, and R_t is the one-period nominal interest rate. Full details and derivations are reported in the Appendix.

In Figure 2.10, we show the results for mortgagors in the left column and those for outright owners in the right column (renters' responses are reported in the Appendix). Note that in this extended version of the model, mortgagors' income still comes from labor only while for outright owners this is the sum of labor and financial income, with the latter stemming from asset returns. The main message from the collateral constraint model in Figure 2.10 chimes with the previous results in Figure 2.9 from the exogenous credit limit model. In particular, it is still the case that (i) the spending responses are heterogeneous across both household groups and goods categories, (ii) the gap in the expenditure changes between the one year (solid red line) and the seven year (broken green line) maturity calibrations is small and (iii) the income responses of households with mortgage debt and outright owners are similar.

The difference between the calibration with a one quarter fixed-rate duration (blue line with circles) and the calibration with a seven year fixed-rate duration can be as large as 25%. On the one hand, this suggests that two economies with a very different mix of adjustable and fixed rate mortgage products may, in principle, exhibit quite different expenditure responses. On the other hand, Figure 10 also reveals that, using two empirically more plausible calibrations for the average effective fixed rate duration of one year for the U.K. and seven years for the U.S., the differences in the expenditure responses to a temporary monetary policy shock tend to be relatively small, consistent with the results in Garriga, Kydland, and Sustek, 2013.[19]

The results in Figure 2.10 suggest that a more detailed specification of the constraint(s) facing indebted households can provide an important amplification mechanism for the effects of monetary policy. While in the Appendix we provide some

[19]More extreme calibrations of the other structural parameters such as, for instance, the degree of price rigidity and the maximum loan to value in the constraint, can generate significantly larger gaps between the one quarter and the seven year duration impulse response functions. This would come, however, at the cost of two counterfactual predictions. First, the magnitude of both non-durable and durable spending would tend to become implausibly large relative to what we report in the empirical analysis. Second, the income responses would display a marked heterogeneity between mortgagors and outright owners. On the other hand, the small difference between fixed rate contracts of one year and seven years is robust to these more extreme parameterizations.

evidence that a financial accelerator mechanism may be at play in both countries, modeling other frictions — such as the transactions costs of accessing illiquid wealth proposed by Kaplan and Violante, 2014 and investigated further by Kaplan, Moll, and Violante, 2015 in a business cycle model with incomplete markets and nominal rigidities — may have the potential to produce an even larger amplification, which could complement the collateral constraint channel.[20] Nevertheless, our key conclusion is that the general equilibrium effect on income, triggered by the behavior of debt-constrained mortgagors, appears quantitatively more important than the direct effect of an interest rate change on debt repayments. This would seem a key feature for any quantitative model of the monetary transmission mechanism to match.

2.6 Conclusions

Does household indebtedness matter for the transmission of monetary policy? And if so, why? Our analysis suggests that it does matter: the reason is that mortgagors tend to hold little liquid wealth relative to their income and exhibit hand-to-mouth behavior despite owning sizable illiquid assets. Consequently, the general equilibrium effects of an interest rate change on income appear quantitatively more important than the direct 'cash-flow' effect on mortgage payments in accounting for the magnitudes of the estimated expenditure responses across heterogeneous debt positions. We reach these conclusions by performing a detailed evaluation of the effects of monetary policy using survey micro data for the United States and the United Kingdom, grouping households by their housing tenure status.

The focus on the U.S. versus the U.K. is motivated by the marked differences in mortgage market structures, with the majority of American contracts featuring longer durations and fixed rates and the majority of British deals featuring shorter durations and adjustable rates. The focus on households with debt versus outright owners is motivated by the close resemblance of these groups to the constrained and unconstrained agents typically found in heterogeneous agent business-cycle models with housing wealth. Indeed, we also report independent evidence that between 40% and 50% of mortgagors in each country are potentially hand-to-mouth, and that the majority of 'wealthy' hand-to-mouth households are mortgagors.

To corroborate the interpretation of our evidence, we lay out a simple heterogeneous agents framework with a credit constraint and multi-period debt. In the model, the credit constraint and the general equilibrium effect on income play a quantitatively more important role than the direct cash flow effect on mortgage payments in rationalizing our empirical findings. This paper therefore highlights a novel mechanism that

[20] In particular, Appendix F shows that, in both U.K. and U.S. data, (real) house prices and mortgage equity withdrawal change significantly following a monetary policy shock.

works through liquidity shortages facing indebted households and extends our understanding of the transmission of monetary policy beyond the conventional intertemporal substitution channel in representative agent models.

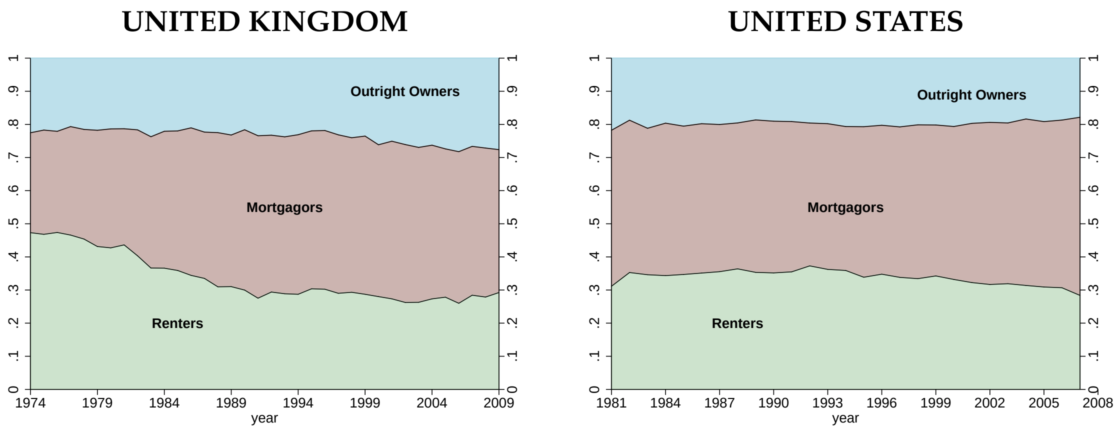


FIGURE 2.1: Share of mortgagors, outright home owners and renters in the U.K. (source: FES/LCFS, 1975-2009) and the U.S. (source: CEX, 1981-2009).

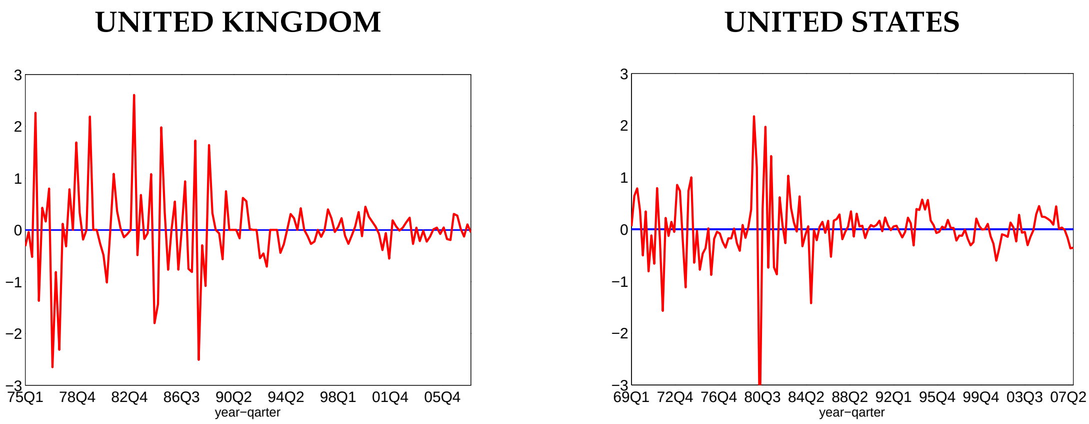


FIGURE 2.2: Monetary policy shocks series. United Kingdom: Cloyne and Huertgen, 2015; United States: updated version of Romer and Romer, 2004.

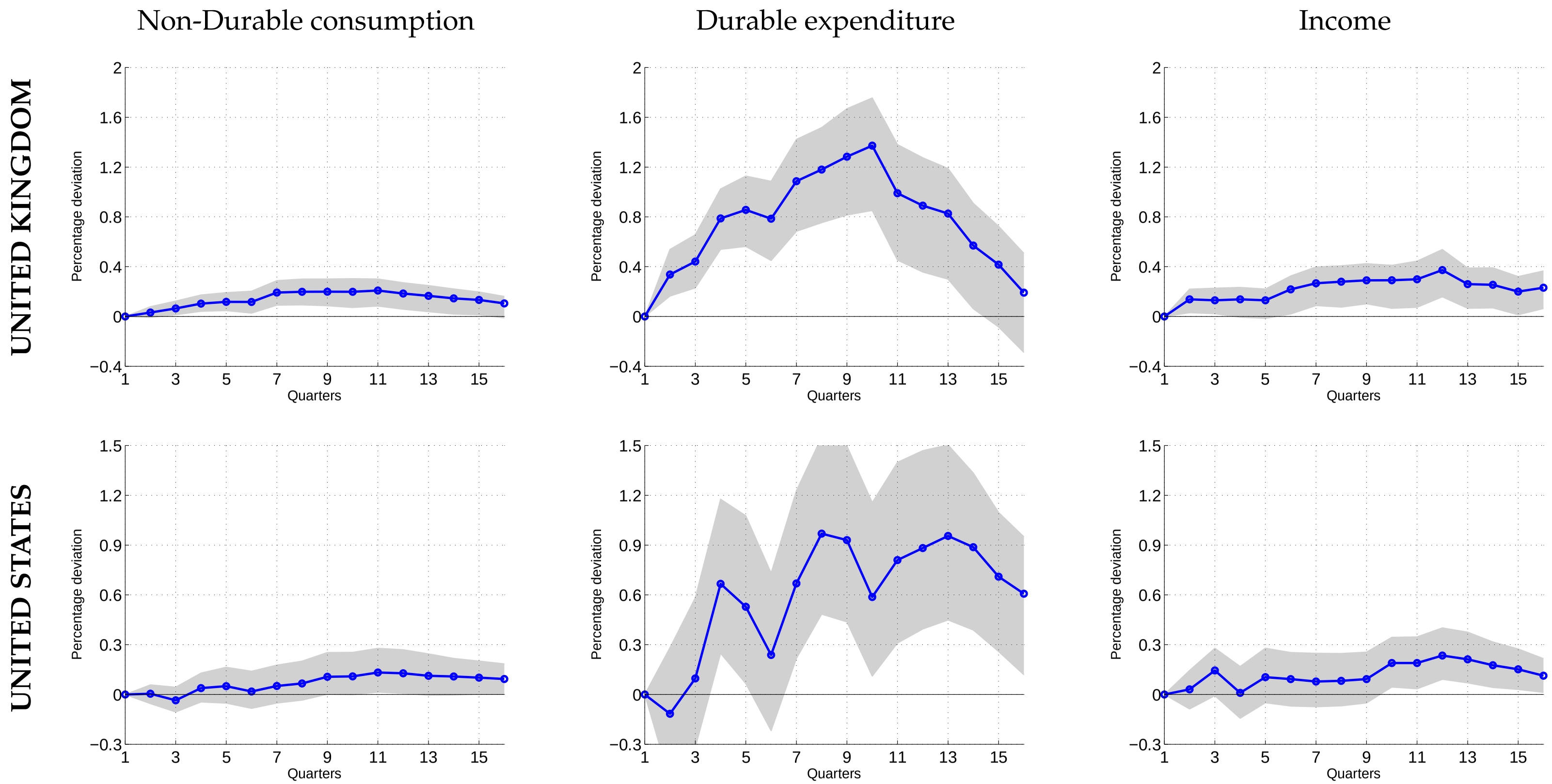


FIGURE 2.3: Dynamic effects of a 25 basis point unanticipated interest rate cut on the consumption of non-durable goods and services, the expenditure on durable goods and household income from official aggregate national statistics. Grey areas are bootstrapped 90% confidence bands. Top row: UK, data range 1975q1-2007q4. Bottom row: US, data range: 1981q1-2007q4.

FIGURE 2.4: Dynamic effects of a 25 basis point unanticipated interest rate cut on the consumption of non-durable goods and services by housing tenure group. Grey areas are bootstrapped 90% confidence bands. Top row: UK (FES/LCFS data). Bottom row: US (CEX data).

FIGURE 2.5: Dynamic effects of a 25 basis point unanticipated interest rate cut on the expenditure of durable goods by housing tenure group. Grey areas are bootstrapped 90% confidence bands. Top row: UK (FES/LCFS data). Bottom row: US (CEX data).

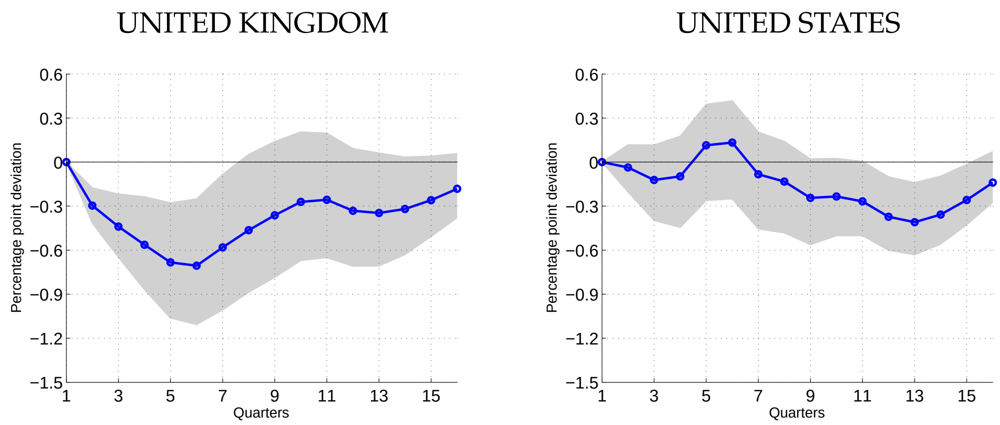


FIGURE 2.6: Dynamic effects of a 25 basis point unanticipated interest rate cut on mortgage payments. Grey areas are bootstrapped 90% confidence bands. Left: UK (FES/LCFS data). Right: US (CEX data).

FIGURE 2.7: Dynamic effects of a 25 basis point unanticipated interest rate cut on income by housing tenure group. Grey areas are bootstrapped 90% confidence bands. Top row: UK (FES/LCFS data). Bottom row: US (CEX data).

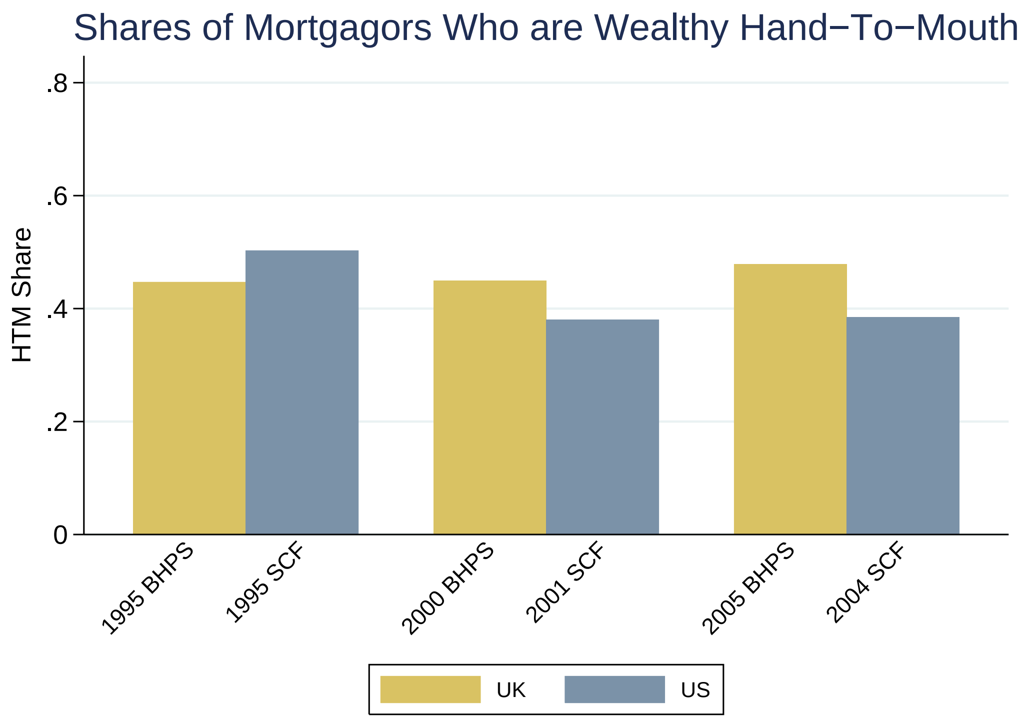


FIGURE 2.8: Shares of Wealthy Hand-To-Mouth (WHTM) mortgagors. U.K. (U.S.) data: 1995, 2000, 2005 waves of the British Household Panel Survey (Survey of Consumer Finances). A household is defined as WHTM if at any given point in time both (i) their net illiquid wealth is positive and (ii) their net liquid wealth is less than half of their total monthly household labor income.

TABLE 2.1: CUMULATIVE CHANGES OVER FOUR YEARS IN US$

Panel A: United Kingdom

	non-durable consumption	*durable expenditure*	*mortgage payments*	*after-tax income*
Mortgagors	308.3	292.3	**-166.4**	695.9
	[112.8, 516.1]	[189.2 , 369.0]	**[-272.2 , -41.7]**	[186.5 , 1105.1]
Outright Owners	-62.6	46.5		451.7
	[-148.2 , 77.4]	[-24.6 , 107.6]		[122.5 , 797.2]
Renters	155.3	19.0		397.3
	[17.9 , 261.8]	[-36.5 , 62.9]		[94.2 , 596.1]

Panel B: United States

	non-durable consumption	*durables expenditure*	*mortgage payments*	*after-tax income*
Mortgagors	305.8	229.3	**-56.3**	757.3
	[58.3 , 554.3]	[122.0 , 350.8]	**[-112.8 , -4.3]**	[196.8 , 1302.0]
Outright Owners	-72.3	54.8		585.3
	[-324.8 , 186.0]	[-10.5 , 127.8]		[83.3 , 1012.8]
Renters	223.3	123.5		439.3
	[32.3 , 412.3]	[30.3 , 213.8]		[112.8 , 699.8]

Note: The Table reports point estimates and bootstrapped 90% confidence bands for the cumulative response (over the 16 quarters of the IRF) of the quarterly values of the variable of interest in 2007 US dollars, following an unanticipated 25 basis point cut in the policy rate. The magnitudes are per household averages.

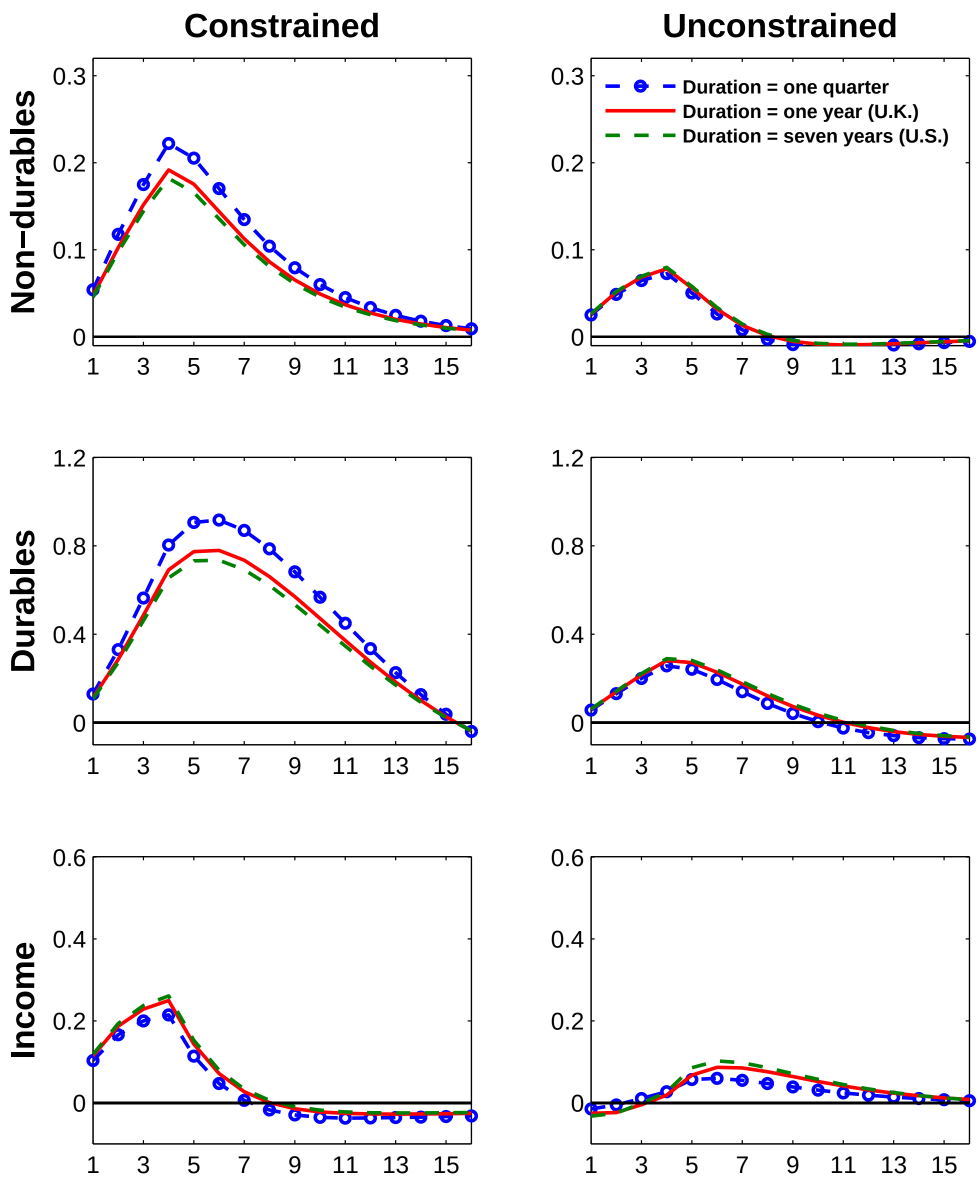


FIGURE 2.9: Response of non-durable consumption, durable expenditure and income in the exogenous debt limit model: constrained vs. unconstrained agents. Duration refers to the effective duration of fixed-rate mortgage contracts in the aggregate economy, which on average is about one year for the U.K. and 7 years for the U.S.

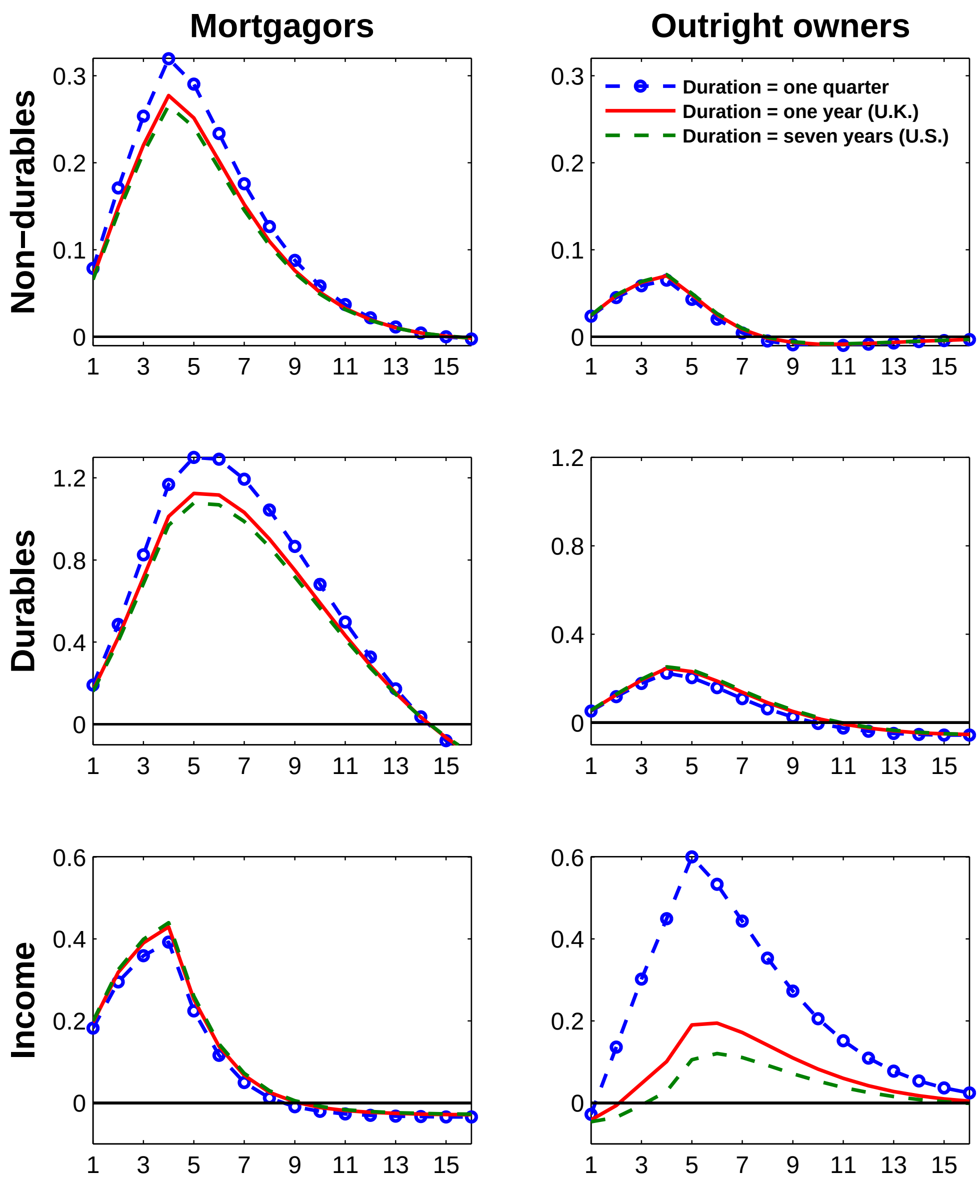


FIGURE 2.10: Response of non-durable consumption, durable expenditure and income in the housing collateral constraint model: mortgagors versus outright owners. Duration refers to the effective duration of fixed-rate mortgage contracts in the aggregate economy, which on average is about one year for the U.K. and 7 years for the U.S.

Chapter 3

Monetary Policy and The Mortgage Market: Evidence from Loan-Level Data

Keywords: mortgage market, first-time-buyer, monetary policy

3.1 Introduction

In Cloyne, Ferreira, and Surico, 2016 we argue that almost 50% of mortgagors in the UK and the US have positive housing and illiquid wealth[1], but are *liquidity constrained* in the sense that liquid assets amount to less than half the monthly (net) income of the household. The consequence of this is that, after an unexpected decrease in short term interest rates, mortgagors consumption of durable and non-durable goods / services increase significantly more than the consumption by outright homeowners (i.e. homeowners without mortgage debt). We show that, although the *direct* change in cash flows due to the nature of variable rate mortgages might explain part of the consumption expenditures increase, most of such increase over a window of 3-4 years seems to arise from *general equilibrium effects* that affect he household's income stream.

The above conclusions, however, emerge from analyzing an *average* mortgagor, both in the data and in a model. This clearly hides differences between households with a mortgage which might turn out to be relevant to understand the transmission of shocks into household decisions and macroeconomic aggregates: whether a mortgagor is able / willing or not to re-mortgage after a number of years paying back the original mortgage (and therefore accumulating equity), or whether a household is a first-time-buyer (FTB) who has saved a minimum deposit, can provide key information about the household's balance sheet position. If there are reasons to believe that the permanent income hypothesis fails in the data, then balance sheet positions will affect decisions about consumption and saving. As recently argued by, among others, Auclert, 2015b and Berger et al., 2015, the *joint distribution* of marginal propensities to consume (MPC) and balance sheet positions (including net housing equity) will then determine the

[1] Illiquid wealth is composed of pension funds, life insurance, and other relatively illiquid investments.

transmission of shocks (such as interest rate or house price) into aggregate variables such as consumption and output.

In this chapter I do two things. Using data on more than 5,000,000 individual mortgages issued in the UK from 1981q1 until 2007q4, I first provide novel evidence on the evolution and behavior of the mortgage market as characterized by the leverage and afford-ability positions of households when mortgages are originated. The data only covers mortgage origination, and the same household is *not* followed in time (i.e. data are repeated cross-sections). In order to analyze the time evolution of household and contract characteristics, I look at the evolution of different percentiles of the *conditional* distributions for the variables of interest (LTV, LTI, real income, age). The distribution of *individual* loan-to-values (LTV) has remained pretty stable over time, while loan-to-income (LTI) ratios at origination have increased by more than 50% in the same period, tracking pretty closely the evolution of house prices. Furthermore, LTIs are also significantly more responsive than LTV to unexpected changes in interest rates. Not surprisingly, there is significant heterogeneity in the evolution and response of LTV and LTI between FTB and non-FTB households. Households that are *first-time-buyers* tend to enter mortgage contracts with much higher LTVs than those that already own a house, have some positive housing equity, and are re-mortgaging. This is of course intuitive, since the latter have already payed part of the principal of their original mortgage when they decide to re-mortgage. I find that after an unexpected decrease in the benchmark interest rate, those households that are able to re-mortgage with relatively low LTV (leverage) tend to extract part of their accumulated equity *over and above* possible capital gains arising from an increase in the price of their house. This is in line with recent evidence put forward by Mian, Rao, and Sufi, 2013, Mian et al., 2015, Wong, 2015b and Berger et al., 2015 among others. On the other hand, LTV at origination remains unchanged for both non-FTB with high (above 85%) LTV and FTB households. I then argue that these responses to shocks are consistent with a type of general equilibrium models with savers and borrowers, where the latter are constrained by the value of their (housing) collateral, and are able to re-mortgage. These are the same models which Cloyne, Ferreira, and Surico, 2016 show do a good job in replicating the behavior of non-durable consumption, durable expenditures and disposable income observed from household level data in the UK and the US. The mechanism through which the model replicates the empirical findings is the following. Relatively impatient households are required to use housing as collateral when borrowing. If the desired borrowing is higher than their maximum collateralizable value of their real estate, then their current consumption expenditures will be determined by current labor income, as well as current changes in house prices. A contractionary monetary policy shock that triggers a decrease in house prices (given the nominal short-term rigidities in the goods market), forces a constrained household to deleverage 1-to-1, while optimally adjusting hours worked. An expansionary monetary shock (i.e. a decrease in the benchmark interest rate) that triggers an increase in house prices, allows a household with positive housing equity to either (i) use the extra equity to increase borrowing and current consumption 1-to-1, or (ii) save part of the increased housing value as a precaution for future expected consumption needs and negative income shocks. This implies a potential asymmetry in the individual and aggregate response to monetary policy shocks. The monetary policy shock I identify in the data is relatively small, with a standard deviation of around 30bp in the period 1975-2007. By construction (i.e. given the estimation methodology), the estimated impulse response functions (irfs) for LTV and LTI at origination are symmetric. Although the size of the identified interest rate shocks doesn't seem large enough to make the desired borrowing or equity extraction slack, there is not sufficient data to test this empirically. I therefore go back to the structural model and adapt the methodology proposed by Guerrieri and Iacoviello, 2015b in order to approximate the model solution locally while capturing the occasionally binding nature of the collateral constraint. For a standard parametrization of the model economy, including the interest rate rule, numerical exercises seem to confirm the intuition: the interest rate shock considered, parametrized in order to match the data, is not large and/or persistent enough to make the collateral constraint slack. Borrowers then extract all available equity for current consumption, leaving the observed LTV unchanged after a shock, while labor income does not react as much as the value of the stock of debt. These results are in line with previous empirical and model-estimates of responses to monetary policy shocks, as in Christiano, Trabandt, and Walentin, 2010 and Guerrieri and Iacoviello, 2015a.

The remaining of this chapter is structured as follows: in section 3.2, I present the mortgage market data and institutional characteristics for the U.K; section 3.3 presents the main empirical results; section 3.4 then contrasts such results with a relative standard general equilibrium model with collateral constraints; finally, section 3.5 concludes.

3.2 Mortgage Origination in the U.K.

3.2.1 Basic Institutional Characteristics

As it has been well documented elsewhere[2], the evolution of the mortgage market in the UK has some particularities. First, and starting in the early 1980s, a series of deregulation measures introduced significant competition and *laissez-faire* in the market for residential mortgages, which until then was dominated by a cartel of Building Societies. Among other important measures, several policies introduced the use of credit scoring techniques (1982), allowed lenders to set interest rates without a "recommendation" from the Building Societies Association (BSA) (starting in 1984), and made it easier for lenders to access wholesale funding and to choose the composition of their

[2] See, for example, Besley, Meads, and Suirco, 2012, Best et al., 2015 and Cloyne, Ferreira, and Surico, 2016.

loan book (starting in 1986 with the Building Society Act). These limits and restrictions have been steadily updated until these days. Second, and for reasons which are still being debated (see the discussion in Miles, 2005), residential mortgages originated in the U.K until right before the starting of the financial crisis where mostly of two types: (i) a fully adjustable rate type (ARM), meaning that the interest rate payments are usually adjusted with changes in the Bank Rate set by the Bank of England and the corresponding changes in the funding costs of lending institutions; and (ii) a mortgage type with a rate that is fixed for 2 to 5 years, after which the rate is "reset" and becomes variable with a high mark-up over the funds rate. These prevalent mortgage types are very different from the prevalent ones in the US.[3]

3.2.2 Data

I use information on *individual* mortgage origination for the U.K. at a quarterly frequency between 1981q1 and 2013q4.[4] The data comes from two main sources: (i) between 1981q1 and 2005q1, I use a 5% sample of all mortgages originated each quarter and collected by the Survey of Mortgage Lenders (SVL)[5], and (ii) after 2005q2, the *universe* of mortgage origination / product sales as collected by the Financial Conduct Authority (FCA). Both sample and universe contain basic information on the household and the completed mortgage product[6]. In particular, I retrieve the following common information for all quarters in the period of analysis: (i) data on product information: loan size, issuance date, property valuation at the time of issuance, initial interest rate; (ii) data on borrower information: age, gross income on which the mortgage is issued and whether the household / borrower is a first-time-buyer (FTB) or a non-first-time-buyer (Non-FTB).[7] . Importantly, households in the data-set are *not* followed in time; i.e. I only have access to repeated cross sections of mortgage origination.

The final database I use presents a distribution for each quarter of, among other variables, loan-to-values (LTV) and loan-to-income (LTI) from individual mortgage deals at origination. The average share of the fixed-rate (2-5 years) products for the period 1981-2007 has fluctuated around 40%-50%.[8] Table 3.1 provides some descriptive statistics on the full sample (1981-2007), while table 3.2 provides statistics for two separate periods: before and after 2001q1. The reason for such split is that around this

[3]For a detailed description of the mortgage markets in the two countries see, among others, Campbell, 2013 and Calza, Monacelli, and Stracca, 2013.

[4]Data is available from 1975 onwards; however, for reasons related to the market structure back in the 70s as describe above, I start my analysis from 1981.

[5]The survey was previously carried out by the Survey of Building Society Mortgages (SBSM).

[6]Mortgages used in a buy-to-let scheme or a comercial property are excluded.

[7]More information was added into the surveys throughout the period. For example, from 1997 onwards, the survey started providing information on the specifics of non-FTB, namely whether the household is moving home, buying a second property, or simply remortgaging (as well as the reasons for rre-mortgaging). Relevant information omitted by the survey and the FCA data are, for example, whether the mortgage is of fixed or variable rate, the credit score assigned to the borrower / loan. More information on the data can be found at the Council of Mortgage Lender's website, http://www.cml.org.uk/home/, as well as at the Product Sales Database of the FCA, http://www.fca.org.uk/firms/systems-reporting/gabriel/help/product-sales-data.

[8]Since 2008, this has radically changed, with fixed-rate mortgages becoming the predominant type.

date, house prices and LTI at origination started a steep increase, as shown in figure 3.3, which lasted until the beginning of the financial crisis.

The main differences between the pre- and post- periods can be seen in terms of the average age and down-payments (in absolute and relative to real income) of mortgagors. Households that where able to get a mortgage before house prices started to increase around 2000-2001, tended to be younger and could afford a house of a given quality with a lower down payment than those that got a mortgage after 2001. This is intuitive: as house pries increase sharply, if banks don't relax credit conditions at the same pace, then a household needs a higher down payment to access a house of a given quality. If such household is a first-time-buyer, then it will need to have saved for a longer period in order to accumulate the required deposit. Indeed, the consequences of increasing house prices can be seen in figure 3.1 which shows the evolution of the composition of mortgagers, i.e. the shares of FTB and non-FTB across time. Starting at the end of the 90s, the share of FTB decreases significantly. Given that the total number of mortgagors has not changed dramatically, this implies a decrease in the number of FTB too.

Pricing and availability of mortgage products bu U.K. banks can be (informally) described as having some sequential aspect to it: First, most banks restrict, either explicitly or implicitly, the loan size and *avilability* according to, among other things, a certain LTI ratio requirement. Second, and conditional on being cataloged within "creditworthy" limits, mortgage products are offered with an initial interest rate which varies in "notches" at different LTV values. In particular, the interest rate offered changes at pre-specified LTVs: 60%, 70%, 75%, 80%, 85% and 90%. In between such notches, the (average) interest rate is almost constant, although the exact "location" of these notches differs by bank and particular product.

3.2.3 Mean and Marginal Mortgagors

Figure 3.2 plots the evolution of the mean (left panel) and 90th percentiles (right panel) for the LTV and LTI distributions in the period 1981q1-2013q4. The figure uses data for *all buyers / borrowers*. Given the menu of contracts that banks and building societies tend to offer in reality, which are usually characterized by a maximum LTV offered, we can thin of the th percentile of the LTV distribution as the *marginal* mortgage originated. This will become clear in section 3.

A first interesting feature emerges: while both the mean and the 90th percentile of the LTV seems to barely move throughout the period (except for the sub-period starting in 2008), the mean (90th percentile) of the LTI distribution almost doubles. In particular, and as seen from figure 3.3, the evolution of the higher percentiles of the LTI distribution seem to track the steep upward movement in house prices since 2001, as measured by the Halifax House Price index (HHPI) and the NationWide House Price

Index (NHPI).[9] This pattern of LTVs and LTIs is not unique to the U.K.. Among other recent works, Campbell and Cocco, 2015 present similar evidence for the U.S. (see figure 1 in their paper). Their ratios, however, and unlike the evidence I am presenting here, are constructed from*aggregate series* for household debt, income and house prices. In addition, the data they use on household mortgages does not include second mortgages (or re-mortgaging), which, as mentioned above and shown in figure 3.1, tended to gain importance once house prices started to increase in the early 2000s. Although before 1997 information on the different types of non-FTB was not available from the surveys, it can be hinted that before 2001, 30% of mortgage origination each quarter corresponded to FTB households; 15%-20% corresponded to households remortgaging, and the other 50%-55% corresponded to other non-FTB (movers, second home buyers, etc). The evolution post-2001 shows a sharp decrease in the share of FTB, which was taken-up by homeowners re-mortgaging. This is intuitive: for a given supply and pricing of mortgages, as soon as house prices started to increase rapidly, renters that had been saving for a deposit having a particular house size / quality in mind, saw their chance to acquire such house significantly reduced (or the cost in terms of current consumption significantly increased). On the other hand, homeowners that had already payed back part of their mortgage saw the value of their corresponding housing equity increase sharply. This created incentives to re-mortgage either in order to extract part of such equity increase, or in order to get a better rate deal assuming lenders priced differently mortgages with different initial LTVs (there is ample evidence for this, on which I comment below).

3.3 Response to Interest Rate Shocks

The information on individual mortgages, though quite granular, represent *outcomes* of an equilibrium between household's consumption-saving decisions and supply / pricing decisions by banks. In this sense, movements in the observed distributions of LTV's and LTI's over time contain changes in both demand and supply forces.

Notwithstanding these confounding forces behind the observed outcomes, analyzing how LTVs and LTIs change after an unexpected movement in interest rates (intertemporal consumption price for households and cost of funding for banks) can still shed light on the transmission of shocks through the mortgage market. With this in mind, I estimate the following empirical model

$$X_{i,t} = \alpha_0^i + g\left(trend; \boldsymbol{\alpha}^i\right) + B^i(L)X_{i,t-1} + C^i(L)S_{t-1} + D^i(L)Z_{i,t-1} + u_{i,t} \tag{3.1}$$

where $X_{i,t}$ can be both the mean and 90th percentile of the distribution of LTV and LTIs at origination (at time t); in other words, $i \in${mean LTV, mean LTI, 90th lTV, 90th

[9] The HHPI includes information on all residential property sales in England and Wales going back to 1983 that are sold for full market value and are lodged nk for registration. The NHPI use similar statistical methods to construct the index, goes back longer than the HHPI, but uses a representative house within the *newly built*.

LTI}.[10] The monetary policy shocks are denoted by S and Z is a vector with additional controls, including quarterly dummies. The $g\,(trend; \boldsymbol{\alpha})$ term represents intercepts and coefficients on a time trend polynomial, with a break in 1993 for the U.K. due to the change towards an inflation targeting monetary policy, as well as a break in 2000q4 when using LTI data.[11]

The usual macroeconomic reverse causation problem arises: the mortgage market, both in terms of demand by households and supply of funds by lending institutions, responds to movements in monetary policy, but monetary policy also responds to developments in household consumption as well as lending and investment. Therefore, regarding unanticipated changes in the short-term interest rate, I use a shock series S_t constructed for the U.K. by means of a "narrative" identification. This method regresses the change in the Bank Rate[12] around the policy decision date on a proxy for the information set available to the policymaker just prior to that decision. This information set includes a range of real time indicators and forecasts to reflect the forward-looking nature of monetary policy.[13] I restrict the shock series 1981q1 - 2007q4, in order to avoid the period where the Bank Rate hit the effective zero-lower bound (ZLB). I then use the estimated parameters from eq. (3.1) to construct point estimates, as well as associated 90% confidence intervals bootstrapped from 10,000 repetitions, of impulse response functions (IRFs) to a 25bp decrease in the Bank Rate.

3.3.1 From Monetary Policy shocks to Mortgage Rate Changes

As a first check and validation of the shock series, I estimate (3.1) using as dependent variable: (i) an average interest rate for mortgages (new and outstanding) at each point in time, and (ii) a marginal mortgage rate (i,e, rate for new mortgage origination) averaged over all LTV values.[14] Figure 3.4 presents the results. Two reassuring patterns emerge: first, after a 25bp (i.e. 0.25% points) decrease in the benchmark policy rate, both the average and marginal mortgage rates decrease; second, the marginal rate reacts and peaks significantly faster than the average rate: while the former reacts with statistical significance already 2 quarters after the shock and peaks at 0.15% after 3-4 quarters, the average rate starts reacting with statistical significance only after 6 quarters. This second result is intuitive given that the average rate emerges from a

[10] I use a backward-looking moving average of 4 quarters for each series in order to eliminate some of the noise inherent in survey data.

[11] The orders of the lag polynomials are chosen using an optimal lag length criteria, namely the corrected AIC. In addition, we have experimented with including the contemporaneous value of the shock and with assuming a different type of trend. Results are robust. Standard errors are bootstrapped using a recursive wild bootstrap.

[12] In the UK, contrary to the US, the Bank rate is indeed the effective interbank rate.

[13] For a detailed description of the method and the resulting series, see among others, Romer and Romer, 2004, Cloyne and Huertgen, 2015 and Cloyne, Ferreira, and Surico, 2016. Importantly, Cloyne and Huertgen, 2015 construct a measure for the U.K. employing this methodology and show that it improves on conventional VAR identification methods.

[14] These variables enter equation (3.1) in levels. The data for bith series is put together by the Bank of England, using the same information I use regarding mortgage origination. They can be found here: BoE Statistical Interactive Database: Interest and Exchange Rates Data.

mix between variable rate mortgages, with a rate that tracks the benchmark rate each period, and fixed rate (2-5 years) mortgages.[15]

The negative response of both rates, which is what one would expect, implies that there is a significant pas-through from policy rates to loan rates. This becomes more apparent if we note that, even when the unexpected changes in the policy rate are correctly identified, the impulse responses estimated above are *general equilibrium* outcomes. In other words, they emerge from the interaction of two forces: on one side, banking sector decisions and competition translates part or all of the funding rate decrease into loans rate decreases (for every LTV or down payment, meaning a shift in the supply curve), while on the other side, the potential increase in loan demand by borrower households due to an inter-temporal substitution channel.

3.3.2 Response in LTVs and LTIs at Origination: All mortgagors

Figure 3.5 then presents the results when (3.1) is estimated on LTV and LTI origination data for all buyers. The patterns emerging from the time series in figure 3.2, namely that LTIs at origination have moved significantly more than LTVs over time (both in absolute terms and in % terms), is apparently also present when one looks at the *deviations from a trend*: while the distribution of LTV at origination seems not to move after an interest rate shock, as captured by the IRF at different percentiles, the opposite is true for the distribution of LTIs. the mean, 75th and 90th percentiles of the latter move significantly and in similar magnitude (as % point deviations from the trend).

Again, these are general equilibrium responses. Notwithstanding, some interesting conclusions can be hinted. First, at the *individual loan level*, the change in the loan size at origination is stronger than the change in household income for (new and existing) borrowers. Second, loan sizes are moving proportionally to house prices; in other words, if house prices are moving in response to the change in the policy rate, then loans are moving proportionally to those. The latter result implies, among other things, that those households getting a new mortgage are either (i) putting a higher down payment in absolute terms if they are a first-time-buyer, or (ii) maintaining or extracting housing equity if they are re-mortgaging, depending on whether house prices are moving or not.

Interestingly, as shown by Best et al., 2015, conditional on lender and product features, household characteristics such as age of principal person and gross income, have very little impact on the level and slope of interest rates charged as a function of the LTV at origination.

[15] At first sight, the fact that the marginal rate peaks at 0.15% after 4 quarters following a 0.25% shock seems a bit low and delayed, given what one would expect in terms of pass-through. Recall, however, that I am filtering the original series before estimation using a 4-quarter backward-looking moving average. This both decreases and delays the irf peak relative to what one would estimate using the original series without filtering.

3.3.3 Heterogeneity Within Mortgagors: FTB vs. non-FTB

The above results rely on looking at all households (contracts) pooled together. Mortgagors differ, however, in (at least) on important dimension: some of them are taking out a mortgage for their first time, moving from a rented dwelling into their own home; a second group is composed by those households that already have some positive housing equity, and are getting a second (or more) mortgage, or re-mortgaging. Their balance sheet, as well as their demographic characteristics, can differ significantly, as shown in table 3.3.

The table shows that FTB are younger, have lower real income, and enter a mortgage with higher LTV and LTI. Interestingly, the spread attached to contracts issued to FTB tends to be *smaller* than the one attached to contracts for non-FTB. The reason for this is that loans to non-FTB households tend to be bigger than those of FTB. For a given maturity, bigger loans increase default probabilities. Such pricing behavior is also in line with the results in Besley, Meads, and Suirco, 2012.

Figure 3.6 now plots the mean and 90th percentile of the LTV and LTI distributions for FTB and non-FTB *separately*. An interesting feature is worth noting: while the LTV and LTI distributions for FTB seem to be quite tight (at least as measured by the mean and 90th percentile) until the years before 2008, when looking at non-FTB that is only the case for LTI. For most of the sample period, mortgages originated for non-FTB's were characterized by a wide range of initial LTV: the mean LTV for this group is stable around 55%, while the 90th percentile fluctuates between 80% and 90%.

The heterogeneity apparent in figure 3.6 implies that the IRFs in figure 3.5 estimated from eq. (3.1) by pooling FTB and non-FTB data, are potentially misleading. To see this, figure 3.7 plots the LTV distributions that are behind the time series in figure 3.6 (pooling all quarters together). The figure shows the LTV distributions for non-FTB and FTB separately, as well as (the thick line) the distribution generated by pooling the data on FTB and non-FTB together. It is clear that the mean and the 90th percentile of the "pooled" distribution might correspond to different type of households altogether.

The next logical step, then, is to look at IRFs for these two groups of mortgagors separately. To do this, I re-estimate eq. (3.1) for FTB and non-FTB series separately. The estimated IRFs and confidence bands are shown in figures 3.8 and 3.9. Some interesting facts emerge: (i) both the response of different percentiles of the LTV and LTI distributions show some heterogeneity between types of mortgagors; (ii) the heterogeneity is particularly strong *for percentiles below 90* of the LTV's: the response for non-FTB is positive and significantly different from 0, while the response for FTB is not statistically different from 0; and (iii) the response of LTI ratios is economically and statistically significant for both type of households. Of course, one would expect smaller responses, *as a percentage deviation from trend*, for the LTV at origination of FTB, since these are on average higher than those of non-FTB (and both are probably constrained by a number very close to 1). However, the interesting fact is the difference in statistical significance.

The positive response for percentiles between below 90 of non-FTB's LTV is consistent with recent evidence in Wong, 2015b and Best et al., 2015 on the fact that households that re-mortgage after a decrease in interest rates (or after the fixed interest rate period ends) tend to withdraw some of the housing equity and use it for consumption expenditures. Note that the positive response of LTV might be implying that re-mortgagors extract equity *over and above* the one gained from house price increases. One possible reason behind this could be the nature of the fixed-rate mortgages in the UK: those households that remortgage are the ones with a 2-5 years fixed rate mortgage, and, given high transaction costs[16], they tend to do so *almost exactly* when they are supposed to, i.e. mainly at the time that the interest rate is set to change. Therefore, when they do re-mortgage, the equity extraction might be larger than if re-mortgaging were costless. As argued by Best et al., 2015, this same nature of UK mortgages seems to render less plausible a change in re-mortgagors characteristics as the explanation behind the positive LTV response for non-FTB. Of course, one possibility that we cannot rule out is that the type of households that choose a fixed-rate mortgage in the first place (i.e. FTB that go for a fixed rate mortgage) changes when interest rate change.

For the 90^{th} percentile (and above) of the non-FTB's LTV at oirigination, on the other hand, the response is close to 0, while the same percentiles of the LTI exibit still positive responses: almost 4 times as big in peak magnitudes (around 0.06% for LTV, and 0.24% for LTI after a 25bp cut in the interest rate).

3.4 LTVs and LTIs in a Model with (Re-)Mortgagors

In this section I propose a relatively standard general equilibrium model with savers and borrowers facing credit constraints, and firms facing nominal rigidities. The goal is to understand whether the transmission of monetary policy shocks generates a pattern in leverage and income in line with the empirical behavior of LTVs and LTIs estimated in the previous section. The framework below is a simplified version of the one used in Cloyne, Ferreira, and Surico, 2016.[17] The focus of the analysis will be on the behavior of borrowers who, in the equilibrium I will analyze in simplified set-up, can be mapped to

[16]For example, search costs for an appropriate new mortgage, mortgage booking fees and early repayment fees.

[17]It builds on the original set-up with savers-borrowers proposed by Kiyotaki and Moore, 1997 and Mankiw, 2000, where relatively impatient households face a credit constraint and can use real assets as collateral for borrowing. This framework was extended by Iacoviello, 2005 to analyze the response of consumption to transitory monetary policy shocks, by assuming that firms face costs to adjust prices; more recently it was used by Eggertsson and Krugman, 2012 to study the interaction of household debt, deleveraging and the zero lower bound in nominal interest rates after a shift in the borrowing limits faced by households. In the paper, we modify the standard framework with three ingredients which we argue are relevant to map the model into the data, and help to generate the heterogeneity in consumption expenditures and income responses that we find using detailed household level data. First, we differentiate between non-durable, durable and housing consumption. Second, we allow for an endogenous housing tenure decision: each household can decide whether to rent housing from others, buy housing with a mortgage, or buy housing outright; households that own housing stock can also choose whether to rent to others. Third, we introduce mortgage products (bonds) with durations longer than 1 period (quarter). Importantly, none of these three ingredients is crucial for the arguments and results below.

non-FTB with high leverage in the data. As seen from figure 3.1, this group represents around 70% of new mortgage contracts issued each quarter, with such share increasing to almost 90% at the peak of the house price boom in the UK. In addition, and although the version of the full model presented in Cloyne, Ferreira, and Surico, 2016 allows for mortgages of varying duration and length of fixed interest rates, I will focus on flexible rate mortgages (i.e. 1-period mortgages). As it is well known[18], these type of mortgages represent around 75% of all mortgages issued before 2007. This translates to an average duration of the fixed-interest period of 1 year.

There are two types of households which are ex-ante heterogeneous: a share ω_{PH} of patient households (PH) and a share ω_{IH} of impatient households (IH). Their discount factors satisfy $0 < \beta^I < \beta^P < 1$. In equilibrium, as shown below, impatient households will borrow from patient ones, so I will denote them as *borrowers* and *savers* respectively. We assume that impatient (patient) households can trade a complete set of arrow securities with other impatient (patient) households, but not with the patient (impatient) households. This implies that idiosyncratic risk can be perfectly shared within households of the same type, but other risks cannot be insured with households of a different type. All households derive utility from the consumption of non-durable goods C_t, services from their *stock* of durable goods, V_t and services from their stock of housing $\tilde{h}_t$, as well as (dis-) utility from hours worked L_t. The lifetime utility is given by

$$U_0^i = E_0 \sum_{t=0}^{\infty} \left(\beta^i\right)^t \left(\frac{x_t^{1-\sigma}}{1-\sigma} + j \log \tilde{h}_t - \frac{L_t^{\eta}}{\eta} \right) \quad i \in \{I, P\} \tag{3.2}$$

with $x_t \equiv x\left(C_t, D_t; \theta, \mu\right)$ a Cobb-Douglass function of non-durables and durable stock, C_t, V_t, with share parameter θ and can also incorporate habits μ. Households might also face *convex* cost of adjusting both the durable stock and the housing stock, given by $\Phi^V\left(D_t, D_{t-1}\right)$ and $\Phi^h\left(h_t, h_{t-1}\right)$ (housing adjustment costs are given in terms of non-durables) where D_t are durable *purchases*.I assume the following functional forms for the convex AC:

$$\Phi^h\left(h_t, h_{t-1}; \theta_h\right) = \frac{\theta_h}{2}\left(\frac{\Delta h_t}{h_{t-1}}\right)^2 q_t^h h_{t-1} \tag{3.3}$$

$$\Phi^V\left(D_t, D_{t-1}; \theta_V\right) = \frac{\theta_V}{2}\left(\frac{D_t}{D_{t-1}}\right)^2 \tag{3.4}$$

In the next subsection, I assume 0 adjustment costs, since these are not crucial for the argument.

The durable stock depreciates at rate δ_D and evolves as

$$V_{t+1} = \left(1 - \Phi^V\left(D_t, D_{t-1}; \theta_V\right)\right) D_t + (1-\delta) V_t$$

[18]See for example Calza, Monacelli, and Stracca, 2013 and Besley, Meads, and Suirco, 2012.

Households can only save and borrow by means of a 1-period mortgage.[19] Borrowers face a credit constraint, and they can use their housing stock as collateral[20]:

$$b_t \leq \phi q_t^h h_t \tag{3.5}$$

where b_{t+1} is the real market value of the stock of long term mortgages at the end of period t and q_t^h is the house price in terms of non-durables. ϕ is the *maximum* LTV at origination; below, I treat this as a fixed parameter. The budget constraint of a borrower, in terms of non-durable consumption, can then be written as[21]

$$C_t + q_t^d D_t + q_t^h \Delta h_t + \frac{b_{t-1}}{\pi_{c,t}} + \Phi^h \left(h_t, h_{t-1}; \theta_h\right) = w_t L_t + S_t b_t \tag{3.6}$$

where $S_t = \frac{1}{R_t}$ is the price of a bond with return R_t. For the saver, denoting variables with " ′ "

$$C_t^{'} + q_t^d D_t^{'} + q_t^h \Delta h_t^{'} - \frac{b_{t-1}^{'}}{\pi_{c,t}} + \Phi^h \left(h_t^{'}, h_{t-1}^{'}; \theta_h\right) = w_t L_t^{'} - S_t b_t^{'} + \Pi_{C,t} + \Pi_{D,c} \tag{3.7}$$

where $\Pi_{C,t}$ and $\Pi_{D,t}$ are the profits from the firms that produce non-durables and durables.

Analyzing the model around such steady state implies that the problem aced by an impatient household is to maximize (3.2) subject to (3.6) (multiplier λ_t), (B.2) (multiplier $\lambda_t q_t^v$), (3.5) (multiplier $\lambda_{CC,t}$), non-negativity constraints and a transversality condition. PH maximize (3.2) subject to (B.117), (B.2), non-negativity constraints and a transversality condition.

Since one group of households is more impatient than the other, the equilibrium solution around the steady state implies that IH will borrow up to their limit (3.5). The optimality conditions for the borrower's problem are then

$$\lambda_t = \beta^L E_t \left(\lambda_{t+1} \frac{1}{S_t \pi_{c,t+1}} \right) + \lambda_{CC,t} \tag{3.8}$$

$$q_t^d = \beta^L E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left(U_{V,t} + (1-\delta) q_{t+1}^d \right) \right) \tag{3.9}$$

$$\lambda_t q_t^h \left(1 + \theta_h \frac{\Delta h_t}{h_{t-1}}\right) = \frac{j}{\tilde{h}_t} + E_t \left(\beta^L \lambda_{t+1} q_{t+1}^h \left(1 + \theta_h \frac{\Delta h_{t+1}}{h_t}\right) \right) + \lambda_{CC,t} \phi q_t^h \tag{3.10}$$

$$L_t^{\eta-1} = \lambda_t w_t \tag{3.11}$$

[19] In Cloyne, Ferreira, and Surico, 2016 we consider long-duration mortgages. However, the analysis around a 0-inflation steady state as carried out below is independent from the duration of the assets. For further discussion on this point, see also Auclert, 2015b.

[20] Here I provide no micro-foundations for this contract outcome. However, there are many ways to do so; see for example Kiyotaki and Moore, 1997 and all the literature thereafter.

[21] Here I assume zero adjustment costs both for durables and housing stock.

with $\lambda_t = \frac{\partial U}{\partial C_t}$. Equation (3.8) defines consumption expenditures given that the collateral constraint binds; eq. (3.9) is the optimal decision for durables expenditures; eq. (3.10) is the optimal decision of housing purchases; and (3.11) is the intra-temporal labor decision. The conditions for the savers are similar, with the difference that now $\lambda^{'}_{CC,t} = 0$; therefore, condition (3.8) is replaced by the standard Euler equation

$$\lambda^{'}_t = \beta^H E_t \left(\lambda^{'}_{t+1} \frac{1}{S_t \pi_{c,t+1}} \right) \tag{3.12}$$

The production side is standard. Output is endogenous. There are two sectors, one producing non-durable consumption goods C_t and another producing durable goods D_t. In each sector, a final producer aggregates different intermediate varieties, with elasticities of substitution between varieties given by ϵ_C and e_D . Monopolistic producer of intermediate variety i in sector j, with $j \in \{C, D\}$, uses a simple technology which requires only labor as an input

$$Y_{j,t}(i) = L_{j,t}(i) \tag{3.13}$$

Labor markets are competitive. Each intermediate producer sets price $P_{j,t}(i)$ but faces quadratic adjustment costs a la Rotemberg, 1982, characterized by parameter θ_j measuring the degree of sector j's price rigidity:

$$AC(i)_{j,t} = \frac{\theta_j}{2} \left(\frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t} \quad j \in \{C, D\} \tag{3.14}$$

In the symmetric equilibrium where intermediate producers use the same amount of labor, the evolution of prices in each sector (Phillips curves) in log-deviations from a 0-inflation steady state are:

$$\begin{aligned} \hat{\pi}_{D,t} &= \beta^H E_t \left(\hat{\pi}_{D,t+1} \right) + \left(\frac{\epsilon_D - 1}{\vartheta_D} \right) \hat{mc}_{D,t} \\ \hat{\pi}_{c,t} &= \beta^H E_t \left(\hat{\pi}_{c,t+1} \right) + \left(\frac{\epsilon_C - 1}{\vartheta_C} \right) \hat{mc}_{C,t} \end{aligned}$$

where $mc_{j,t} = \frac{W_t}{P_{j,t}}$ is the real marginal cost in sector $j \in \{C, D\}$.

To close the model Monetary policy is carried out via a simple Taylor rule

$$R_t = (R_{t-1})^{r_R} \left(\pi_{t-1}^{1+r_\pi} \left(\frac{Y_{t-1}}{Y} \right)^{r_Y} \bar{rr} \right)^{1-r_R} \epsilon_{R,t} \tag{3.15}$$

where π_t is the CPI, a composite index of non-durable and durable inflation rates $\pi_{C,t}, \pi_{D,t}$, $Y_t = Y_{D,t} + Y_{C,t}$ is total output in the economy, and $\epsilon_{R,t} \sim (0, \sigma_R)$ is an unexpected monetary policy shock.

Since all production is consumed, and since housing adjustment costs $\Phi^h(h_t, h_{t-1})$ are defined in terms of non-durable goods C, the feasibility constraints are

$$Y_{C,t} = \omega_{IH}\left(C_t + \Phi^h(h_t, h_{t-1})\right) + \omega_{PH}\left(C_t' + \Phi^h\left(h_t', h_{t-1}'\right)\right) + \frac{\theta_C}{2}(\pi_{C,t} - 1)^2 Y_{C,t} \quad (3.16)$$

$$Y_{D,t} = \omega_{IH} D_t + \omega_{PH} D_t' + \frac{\theta_D}{2}(\pi_{D,t} - 1)^2 Y_{D,t} \quad (3.17)$$

Finally, labor, housing and mortgage markets clearing requires

$$\begin{aligned} L_{C,t} + L_{D,t} &= \omega_{IH} L_t + \omega_{PH} L_t' \\ H &= \omega_{IH} h_t + \omega_{PH} h_t' \\ 0 &= \omega_{IH} b_t + \omega_{PH} b_t' \end{aligned}$$

A *competitive equilibrium* in this economy is a set of sequences for C_t, C_t', D_t, D_t', V_t, V_t', h_t, h_t', L_t, L_t', b_t, b_t', Lagrange multipliers and prices $P_{C,t}, P_{D,t}, w_t, q_t^h, q_t^d, S_t, R_t$ such that:

1. C_t, C_t', D_t, D_t', V_t, V_t', h_t, h_t', L_t, L_t', b_t, b_t' , as well as the corresponding Lagrange multipliers, solve the household problems for given prices and interest rate R_t.
2. $P_{C,t}, P_{D,t}, L_{C,t}, L_{D,t}$ solve the firms problem for given w_t
3. R_t is set according to (3.15), and S_t, R_t satisfy a no arbitrage condition.
4. Prices $P_{C,t}, P_{D,t}, w_t, q_t^h, q_t^d, S_t, R_t$ are such that all markets clear.

In this set-up, the steady state interest rate is given by the discount factor of the patient household

$$r^{SS} = \frac{1 - \beta^H}{\beta^H}$$

From the optimality conditions for a borrower, in steady state we have

$$1 = \beta^L R + \frac{\lambda_{CC}}{\lambda}$$

with $\lambda = \frac{\partial U}{\partial C}$. For standard properties of x_t (see below) it is the case that

$$\lambda_{CC} = \left(1 - \beta^L R\right)\lambda > 0 \quad (3.18)$$

indeed verifying that the borrower is constrained in SS.

Near[22] such steady state, then, consumption for borrowers is given by

$$\tilde{C}_t = w_t L_t + S_t b_t - q_t^h \Delta h_t - \frac{b_{t-1}}{\pi_{c,t}} \quad (3.19)$$

[22]See for conditions under which a linearization around this steady state is valid.

where $\tilde{C}_t \equiv C_t + q_t^d D_t$ is total real (in terms of non-durables) expenditure in consumption goods.

3.4.1 Parametrization

For the numerical analysis below, I assume the following functional form for consumption utility:

$$x_t = \left(C_t^{\theta} V_t^{1-\theta} - \mu C_{t-1}^{\theta} V_{t-1}^{1-\theta}\right) \tag{3.20}$$

where $\theta \in [0, 1]$ is a share parameter and $\mu \in [0, 1)$ captures habit persistence. In the bench,ark calibration, I will assume no habits, $\mu = 0$.

Table 3.4 presents the parameter values used in the experiment below. Most of the values are calibrated in a standard way. A couple of parameters deserve some further explanation, though. I calibrate the housing demand shifter j as well as the elasticity for different varieties of intermediate goods (which determines the wage in a steady state) in such a way that, in a 0-inflation steady state the maximum LTV, as well as the LTI, match the 90th percentile of the LTV and LTI at origination, averaged over the period 1981-2007 in the UK; these numbers are 92% and 2.6 respectively. Importantly, given that the model described above abstracts from investment, government spending and next exports, I calibrate the debt-to-output $\frac{SB}{Y}$ in the model to match the *debt-to-expenditure* in the data, which for the UK is around 1.8.

3.4.2 Solution and Results

Linear Approximation

I first log-linearize the optimality and equilibrium conditions of the model around a 0-inflation steady state in which the collateral constraint binds for borrowers, and replicate the 25bp monetary policy shock used to construct the empirical IRFs. Figures 3.10-3.13 present results regarding the response of the LTV and LTI, as well as the real value of debt, house prices and labor income for the *representative* borrower.[23]I carry out two different comparisons. First, I compute IRFs for economies with different values of the housing adjustment cost parameter, θ_h, and a benchmark Frisch elasticity of 2[24]. Figure 3.10 plots the response of the LTV and LTI. and [25] The IRFs are qualitatively similar to what we observe using data on all mortgage origination, and they are intuitive. As shown above, the borrowing constraint is binding for impatient households in such steady state. After a decrease in the short term interest rate, savers are willing to bring non-durable, durable as well as housing consumption forward in time. Given

[23]As mentioned above, Cloyne, Ferreira, and Surico, 2016 argue that a similar environment to this one can match, qualitatively and quantitatively, household consumption and income responses for different housing tenure groups as estimated from micro data in the UK and the US. I therefore abstract from showing other IRFs here.

[24]This is the benchmark value used in Cloyne, Ferreira, and Surico, 2016.

[25]It can be shown that the steady state value of variables in this environment does not depend on the value of the Frisch elasticity of labor supply, $\frac{1}{\eta - 1}$.

goods prices are sticky in the short run, this starts to put pressure on wages and house prices in order for labor and house markets to clear. The impatient borrowers take advantage of both higher wages and higher house prices in order to increase hours worked and, due to the positive value of housing as collateral, also the stock of houses. The adjustment of the housing stock is costly, though, which is reflected in the fact that, as also shown in figure 3.11, the increase in the housing stock (translated into a bigger loan) is smaller when adjustment costs increase. This is reflected into a smaller and more gradual increase in the LTI, approximately matching what we observe in the data. In addition, around the steady state borrowers extract as much equity as possible, given that interest rates are still relatively low. This implies that the LTV at origination remains constant at the maximum available to the household, given by ϕ.

Figures 3.12 and 3.13 present the IRF under different Frisch elasticities of labor supply, for a benchmark value of the adjusting costs for housing of $\theta_h = 0.6$.

Piecewise-Linear Approximation And Asymmetric Responses

The main advantage of the (log-)linear approximation is that it can easily deal with many state variables, as in the current case. However, a natural caveat arises regarding the above simulation results: the the null response of the LTV is indeed an artifact of the local linear approximation around the steady state in which the collateral constraint binds for borrowers. The approximation is valid if the the interest rate shock is "small enough", such that the impatient households still want to borrow up to their credit limit in order to consume or roll over existing debt. In such cases, then, the loan obtained by the borrower is always the largest allowed by their housing collateral value. The same household might, however, decide to de-leverage or not use their full collateral value if the change in interest rates and income is above certain level. For "big" shocks, the linear approximation to the household behavior might be very poor.

In order to capture the possible asymmetric response of variables to shocks due to the nature of the collateral constraint binding only occasionally, I adapt the non-linear approximation method proposed by Guerrieri and Iacoviello, 2015b. The idea behind it is the following. I identify two "regimes", depending on whether the collateral constraint binds or not for the impatient household. Then a first-order approximation to the solution around the *same* point for each regime is used. In the model above, that point is the 0-inflation steady-state where the collateral constraint binds, as shown in equation (3.18). Although the approximated solution is linear for each regime, household decisions and the dynamics of the economy after a shock can be highly non-linear, depending on how long the agents expect the current regime to last, which depends on the current state variables, as well as the size and persistence of the shock.[26] An important drawback of the approximated solution is that, as any linear solution, it discards

[26] A similar approach has been used in the analysis of the zero-lower-bound (ZLB) as a constraint to nominal interest rates; see for example Christiano, Eichenbaum, and Rebelo, 2011.

information about the volatility of future shocks and therefore about precautionary behavior. In appendix section C.1 I sketch the basics of the solution algorithm.

I carry out the following simulation: two sequences of 60 interest rate innovations is fed into the piece-wise linear solution of the model. Each sequence is composed of: an initial sub-period $1, ..., t_1^* - 1$ with 0 innovations, a *negative* innovation in period t_1^*, 0 innovations in periods $t_1^* + 1, ..., t_2^* - 1$, a *positive* innovation in period t_2^* , and 0 innovations in periods $t_2^* + 1, ..., T$, with $T = 60$. Figure 3.14 illustrates the response of LTV and LTI at origination when the size of the negative and positive interest rate innovations in periods t_1^* and t_2^* is 25bp (as in the empirical irfs), while figure 3.15 presents the case where the size of the innovations is 300bp (a change of 3 p.p in the interest rate). This magnitude close the smallest innovation which generates some asymmetry, conditional on the benchmark persistence in the monetary policy rule (3.15) as presented in table 3.4. For comparison, the the behavior of LTV and LTIs under the linear solution are also showed.

The simulations suggest that the behavior portrayed in irf coming out from the first-order approximation is not purely an artifact of the linearity of the solution around the steady state where the collateral constraint binds. Given the empirically motivated size of the monetary policy shock, and the benchmark parametrization of the monetary policy rule, the general equilibrium effects on labor income and house prices are not enough for borrowers to expect current and future collateral constraints to be slack. Again, this approximation ignores the consequences of precautionary behavior. Therefore, the above results cannot rule out the possibility that, for the same shock size that triggers an increase in house prices and parametrization of the monetary policy rule, borrowers decide to deleverage in order allow some room for borrowing in the event of a negative shock in the future.

3.5 Concluding Remarks

I have presented evidence on the behavior of observed leverage (LTV) and affordability (LTI) ratios at the individual mortgage level in the UK mortgage market. Perhaps surprisingly, both the evolution and response to interest rate shocks of the distribution of LTVs at origination appears to be much less volatile than that of LTIs. Although I cannot rule out the possibility of the endogenous selection of households into getting a mortgage affecting the interpretation of these results, evidence form previous papers on the pricing of mortgage contracts point suggest that the mechanism behind the transmission of monetary policy through the mortgage market is worth studying.

I then analyzed potential mechanisms through the lens of a standard general equilibrium model with heterogeneous agents and nominal rigidities, with certain important features that are close to the mortgage market in the UK. Simple numerical exercises which account for local non-linearity in the behavior of households seem to confirm that the patterns estimated from the (albeit imperfect) data are not spurious, and can provide information on the transmission of monetary policy into expenditures and income. Of course, significant work, both empirical and theoretical, still needs to be done on this front. This is part of my ongoing research agenda.

Tables

TABLE 3.1: Descriptive statistics for selected variables. All years 1975q1-2007q4. All mortgagors.

	Mean	**Median**	**St.Dev.**	**p25**	**p75**
Age [a]	32.3	30	8.5	26	37
Real Income [b]	147.80	127.15	87.80	95.52	173.65
Downpayment	116.68	59.38	156.13	19.75	157.70
LTV	0.77	0.83	0.18	0.64	0.94
LTI	2.19	2.16	0.58	1.76	2.57
Spread [c]	2.40	1.68	1.71	1.14	4.00

Note: individual mortgage origination come from (i) the Survey of Mortgage Lenders (SML) between 1975 and 2005q2 (excluding 1978 due to lack of data); and (ii) the Financial Conduct Authority (FCA after 2005q2.
[a] Age reflects the age of the main borrower in the contract. [b] Real values for income and down-payments are computed by deflating nominal values using the consumer retail price index that excludes mortgage interest payments (RPIX). [c] The spread at origination is computed as the difference between the contracted mortgage rate and the average deposit rate for Building Societies.

TABLE 3.2: Descriptive statistics for selected variables. Pre and Post 2001. All mortgagors.

	Mean	**Median**	**St.Dev.**	**p25**	**p75**
	1975q1-2000q4				
Age [a]	32	30	8.4	26	37
Real Income [b]	141.74	124.26	77.62	94.06	167.10
Downpayment	107.26	55.40	139.03	18.83	147.77
LTV	0.78	0.83	0.18	0.64	0.94
LTI	2.13	2.12	0.53	1.74	2.50
Spread [c]	2.53	1.84	1.72	1.25	4.00
	2001q1-2007q4				
Age	34.8	33	9.09	28	40
Real Income	213.36	178.56	145.65	124.82	255.37
Downpayment	218.36	128.77	260.00	40.00	304.88
LTV	0.76	0.80	0.18	0.63	0.92
LTI	2.76	2.78	0.80	2.18	3.27
Spread	0.99	0.83	0.71	0.46	1.40

Note: individual mortgage origination come from (i) the Survey of Mortgage Lenders (SML) between 1975 and 2005q2 (excluding 1978 due to lack of data); and (ii) the Financial Conduct Authority (FCA after 2005q2. [a] Age reflects the age of the main borrower in the contract. [b] Real values for income and downpayments are computed by deflating nominal values using the consumer retail price index that excludes mortgage interest payments (RPIX). [c] The spread at origination is computed as the difference between the contracted mortgage rate and the average deposit rate for Building Societies.

TABLE 3.3: Descriptive statistics for selected variables. All years 1975q1-2007q4. FTB and Non-FTB.

	Mean	Median	St.Dev.	p25	p75
			FTB		
Age [a]	29.2	27	7.6	24	33
Real Income [b]	132.58	117.74	69.91	89.62	155.81
Downpayment	54.33	23.98	92.90	12.10	57.95
LTV	0.87	0.91	0.14	0.83	0.95
LTI	2.22	2.20	0.56	1.81	2.60
Spread [c]	2.34	1.64	1.72	1.03	4.00
			Non-FTB		
Age	34.9	33	8.4	29	40
Real Income	161.08	137.23	98.95	101.88	190.01
Downpayment	171.03	123.61	178.22	52.20	227.60
LTV	0.69	0.70	0.18	0.55	0.86
LTI	2.16	2.12	0.61	1.72	2.53
Spread	2.46	1.74	1.70	1.20	4.00

Note: individual mortgage origination come from (i) the Survey of Mortgage Lenders (SML) between 1975 and 2005q2 (excluding 1978 due to lack of data); and (ii) the Financial Conduct Authority (FCA after 2005q2. [a] Age reflects the age of the main borrower in the contract. [b] Real values for income and downpayments are computed by deflating nominal values using the consumer retail price index that excludes mortgage interest payments (RPIX). [c] The spread at origination is computed as the difference between the contracted mortgage rate and the average deposit rate for Building Societies.

TABLE 3.4: Calibration of the model.

Parameter	Description	Value
σ	elasticity of intertemporal substitution	0.5
β^L, β^H	discount factor: mortgagors, outright owners	0.95, 0.99
$1/(\eta - 1)$	Frisch elasticity of labor supply (benchmark)	2
j	housing demand shifter	0.468
δ	depreciation rate durables	0.025
$\varepsilon_{C,D}$	elasticity of varieties	4
$\vartheta_{C,D}$	cost of adjusting prices	150
r_π, r_Y, r_R	Taylor rule: CPI, output, smoothing	1.5,.05,.6
ω_{IH}	share constrained households	50%
ω_{PH}	share unconstrained households	50%
ϕ	maximum LTV	92%
Sb/Y	debt to expenditure ratio	1.8

Figures

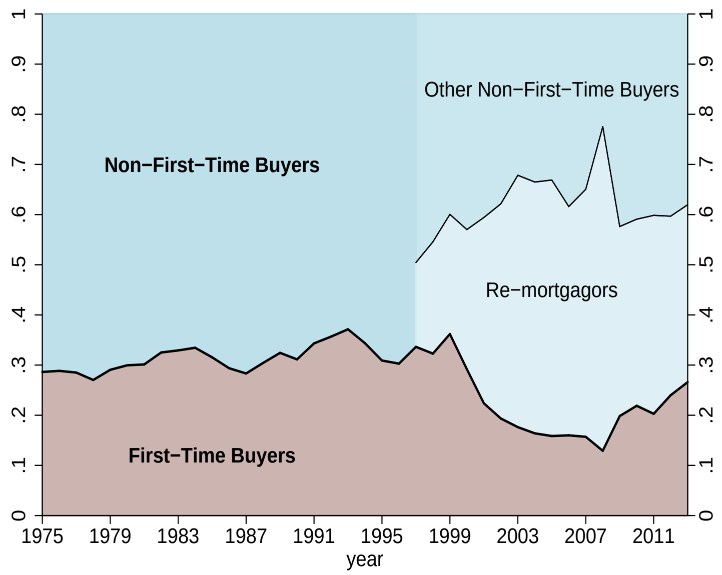


FIGURE 3.1: Evolution of the composition of mortgage origination in the U.K. Annual averages of quarterly data between 1975q1-2013q4. Constructed using data from: the Council of Mortgage Lenders (CML) until 2005q1, and Financial Conduct Authority (FCA) from 2005q2 onwards.

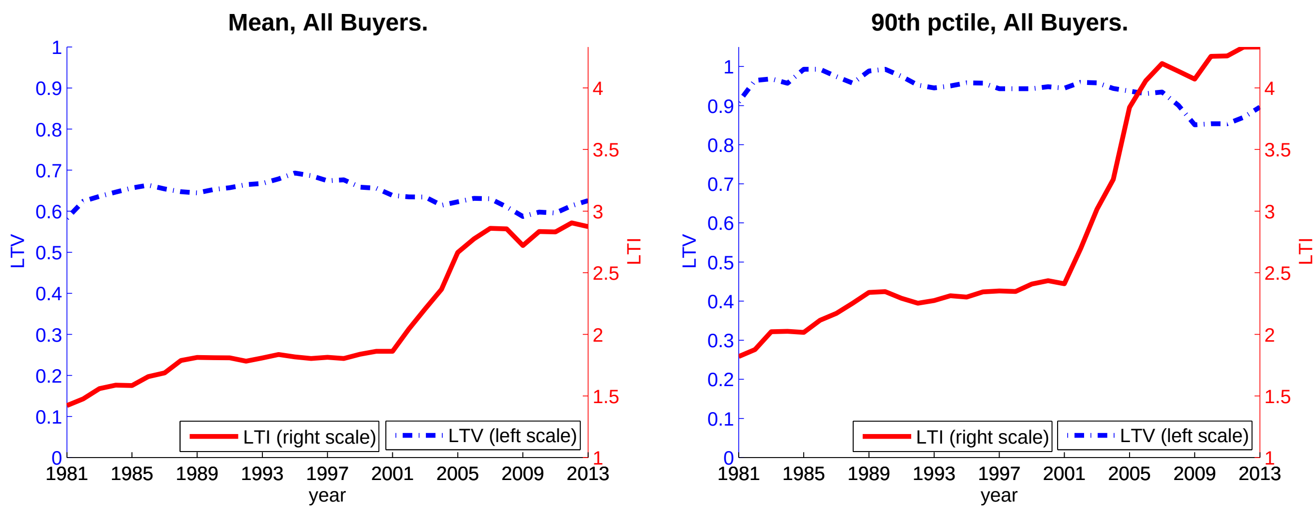


FIGURE 3.2: Evolution of the mean (left) and 90th percentile (right) of the LTV and LTI distributions for mortgage origination in the UK. **All buyers**. Constructed using annual averages of quarterly data between 1981q1-2013q4 from the Council of Mortgage Lenders (CML) and Financial Conduct Authority (FCA).

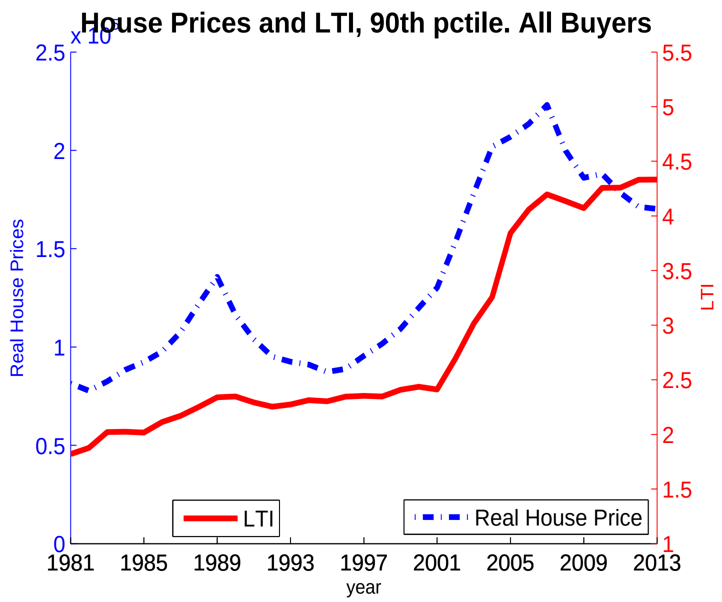


FIGURE 3.3: Real House Prices and 90th percentile of the LTI distribution (all buyers). Annual averages of quarterly data between 1981q1-2013q4 1981q1-2013q4. Constructed using data from: (i) NationWide , and (ii) the Council of Mortgage Lenders (CML) until 2005q1, and Financial Conduct Authority (FCA) from 2005q2 onwards.

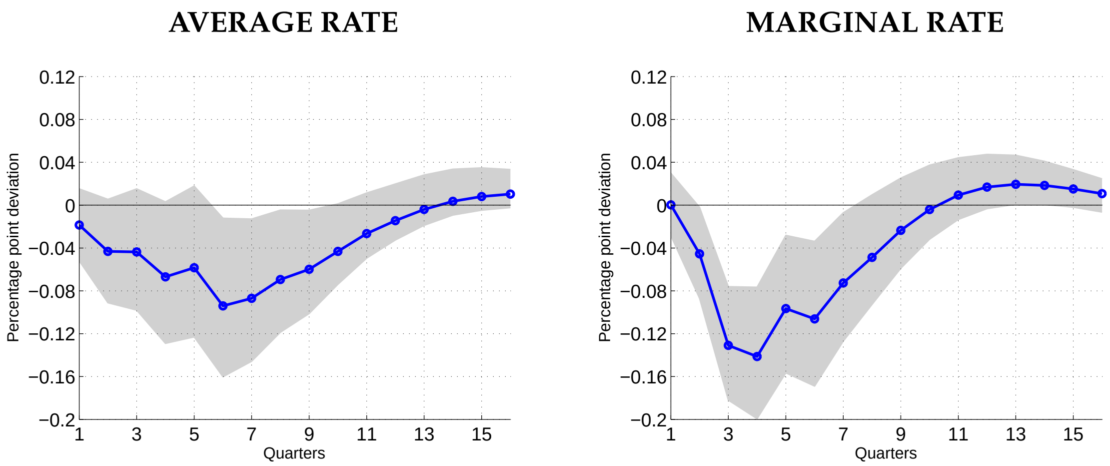


FIGURE 3.4: Impulse response functions (IRFs) to a 25bp decrease in the benchmark (Bank) rate of: the **average effective mortgage rate (left)** on all mortgages, and **marginal mortgage rate (right)** on new mortgages. Point-estimates constructed from (3.1) in the text, estimated on 1975q1-2007q4 data; shaded areas represent 90% confidence intervals bootstrapped from 10,000 repetitions. Data source: Bank of England, Council of Mortgage Lenders (CML) and Financial Conduct Authority (FCA).

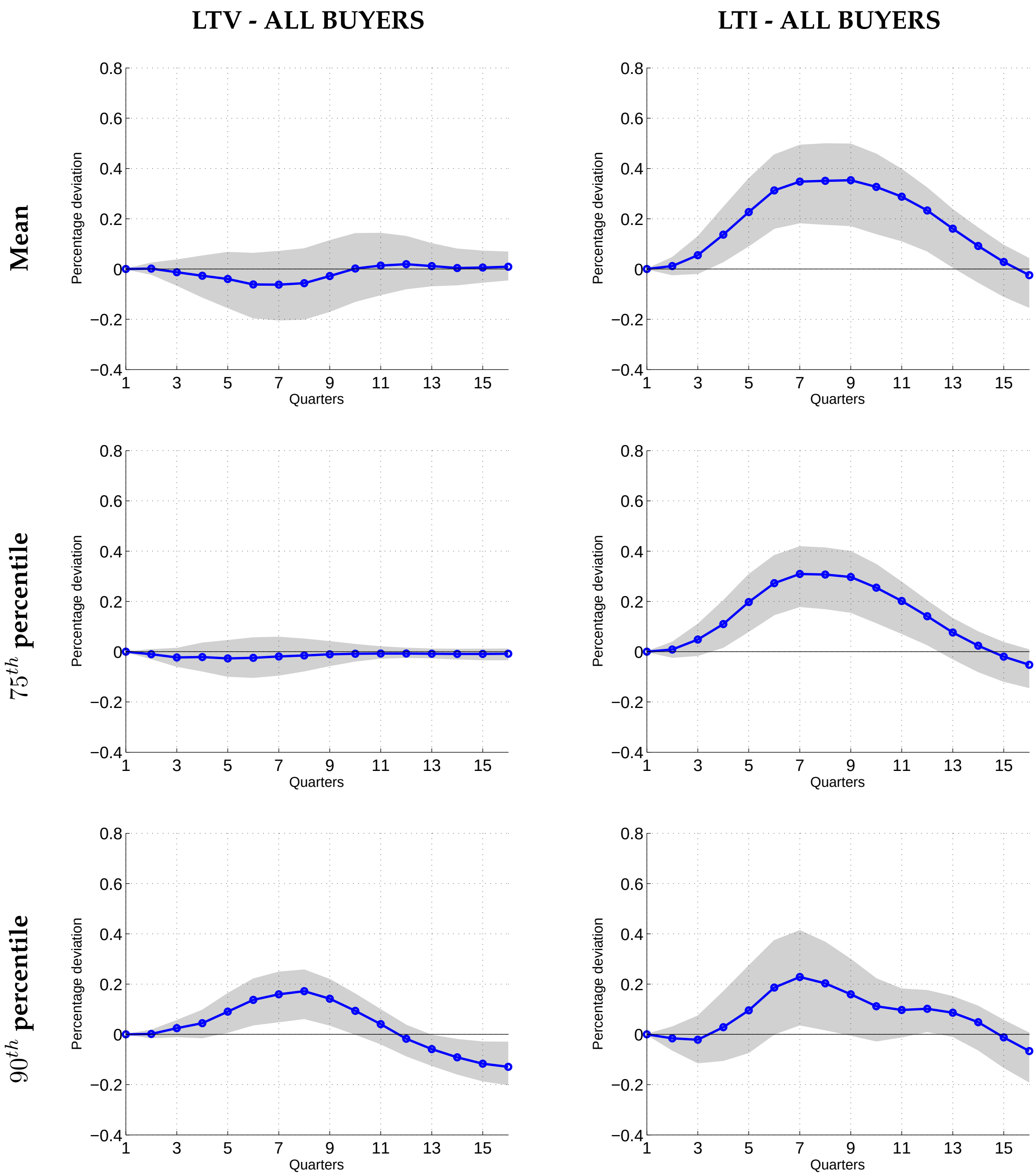


FIGURE 3.5: Impulse response functions (IRFs) to a 25bp decrease in the benchmark (Bank) rate of: the **mean (top row)**, **75th percentile (middle row)** and **90th percentile (bottom row)** of the **LTV (left column)** and **LTI (right column)** distributions at origination for **all buyers**. Point-estimates constructed from (3.1) in the text, estimated on 1981q1-2007q4 data; shaded areas represent 90% confidence intervals bootstrapped from 10,000 repetitions. Data source: Council of Mortgage Lenders (CML) and Financial Conduct Authority (FCA).

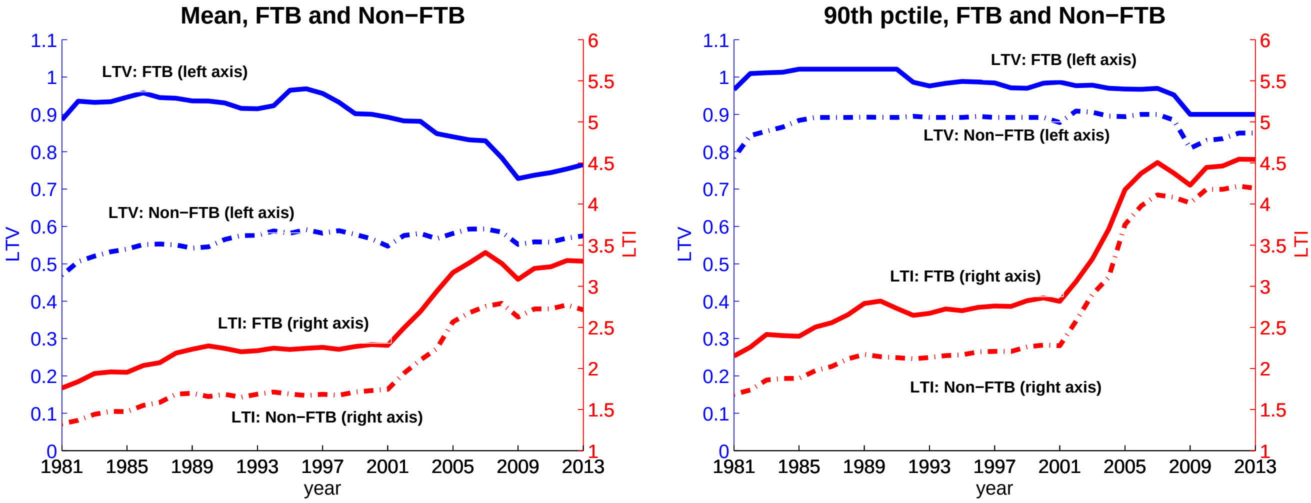


FIGURE 3.6: Evolution of the mean (left) and 90th percentile (right) of the LTV and LTI distributions for mortgage origination in the UK. Separate series for **First-time-buyers (FTB) and Non-First-time-buyers (Non-FTB)**. Constructed using annual averages of quarterly data between 1981q1-2013q4 from the Council of Mortgage Lenders (CML) and Financial Conduct Authority (FCA).

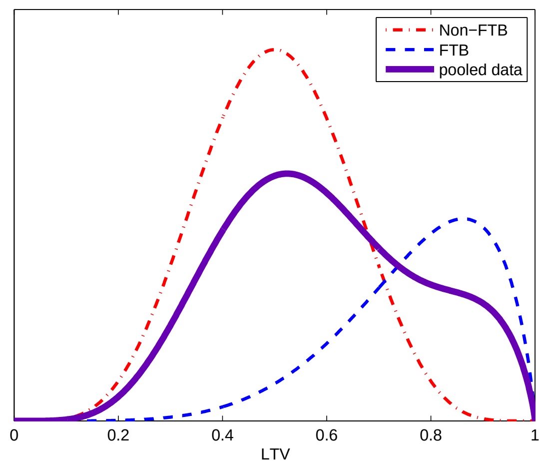


FIGURE 3.7: Confounding LTV distributions at origination

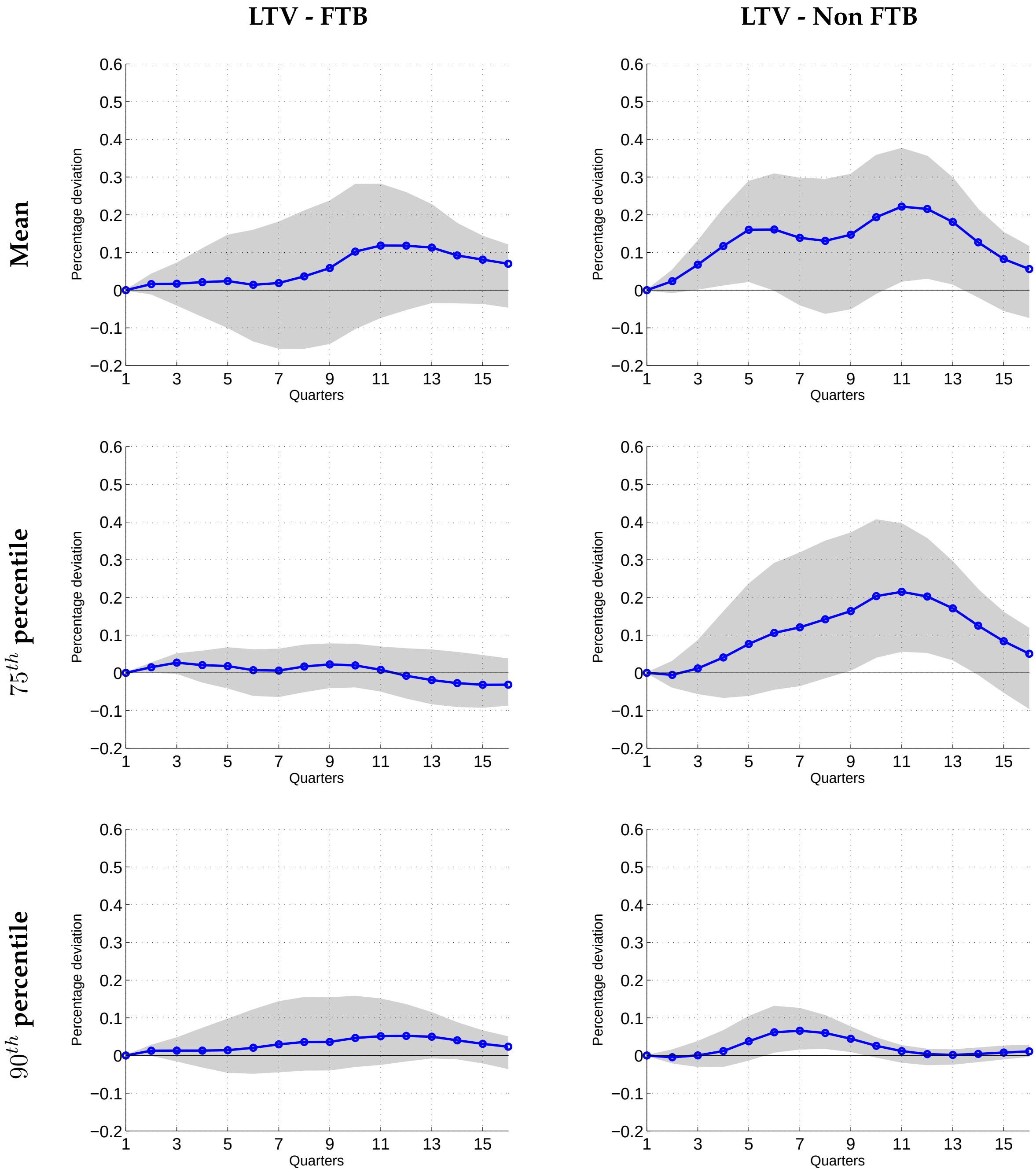


FIGURE 3.8: Impulse response functions (IRFs) to a 25bp decrease in the Bank rate of: the **mean (top row)**, **75th percentile (middle row)** and **90th percentile (bottom row)** of the **LTV** distribution at origination for **FTB (left column) and Non FTB (right column)**. Point-estimates constructed from (3.1) in the text, estimated on 1981q1-2007q4 data; shaded areas represent 90% confidence intervals bootstrapped from 10,000 repetitions. Data source: Council of Mortgage Lenders (CML) and Financial Conduct Authority (FCA).

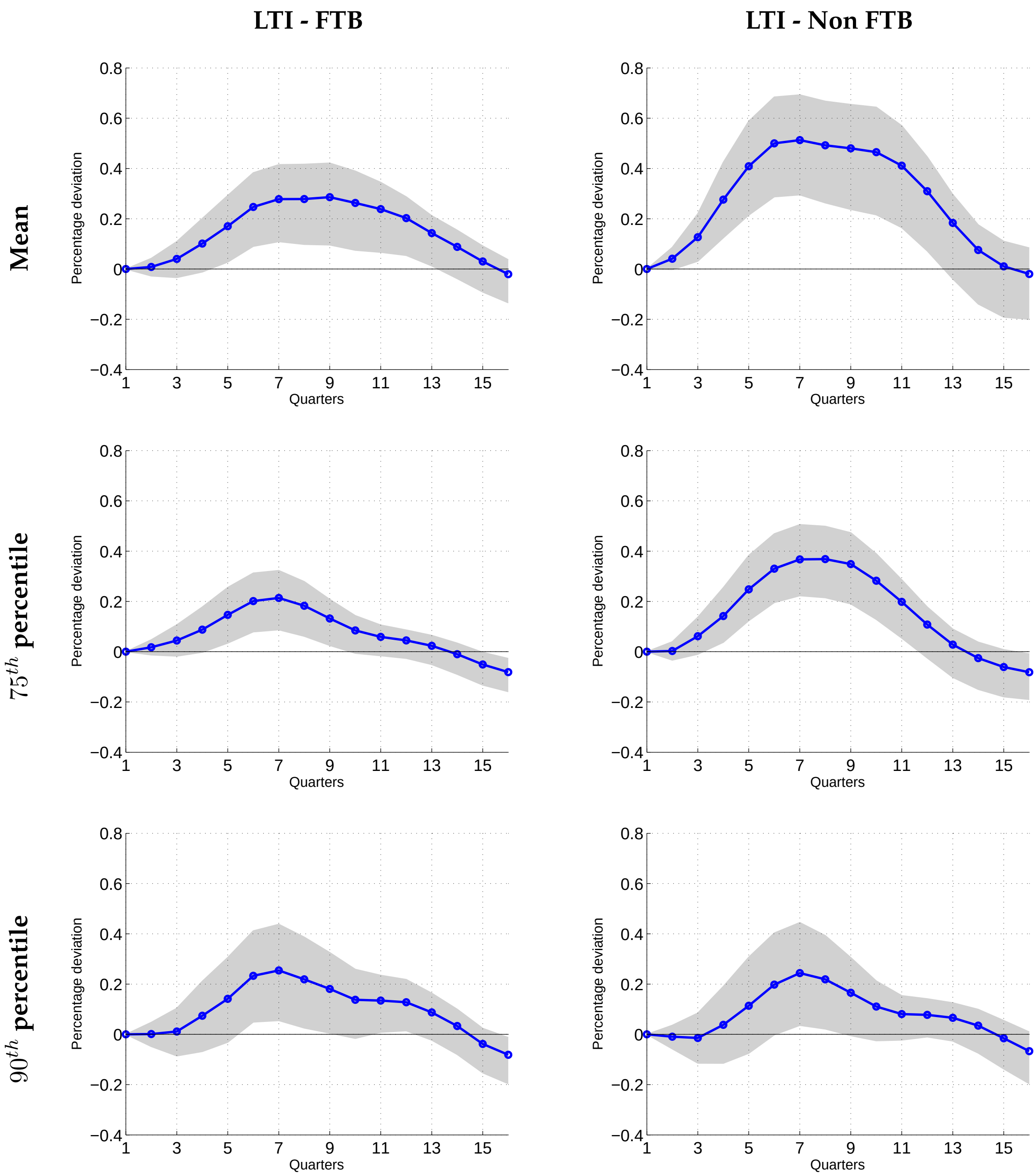


FIGURE 3.9: Impulse response functions (IRFs) to a 25bp decrease in the Bank rate of: the **mean (top row)**, **75th percentile (middle row)** and **90th percentile (bottom row)** of the **LTI** distribution at origination for **FTB (left column) and Non FTB (right column)**. Point-estimates constructed from (3.1) in the text, estimated on 1981q1-2007q4 data; shaded areas represent 90% confidence intervals bootstrapped from 10,000 repetitions. Data source: Council of Mortgage Lenders (CML) and Financial Conduct Authority (FCA).

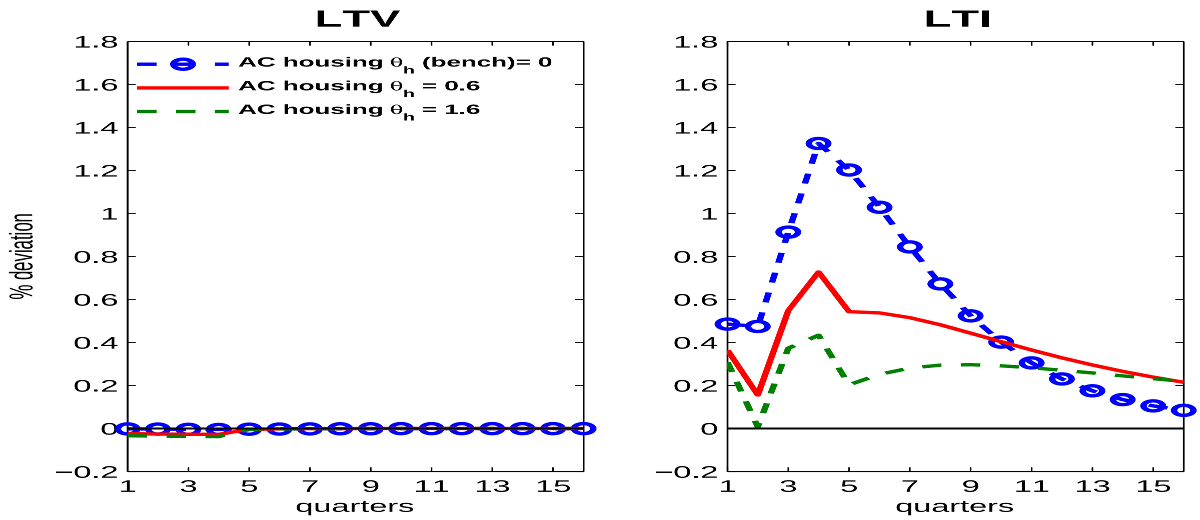


FIGURE 3.10: Mortgagor's LTV and LTI model responses to a 25bp decrease in the interest rate. **Different values for housing adjustment costs. The wage (Frisch) elasticity of labor supply is set to 2.** The LTV in the model is computed as $\frac{S_t b_{t+1}}{h_t q_t^h}$, while the LTI is computed as $\frac{S_t b_{t+1}}{L_t w_t}$.

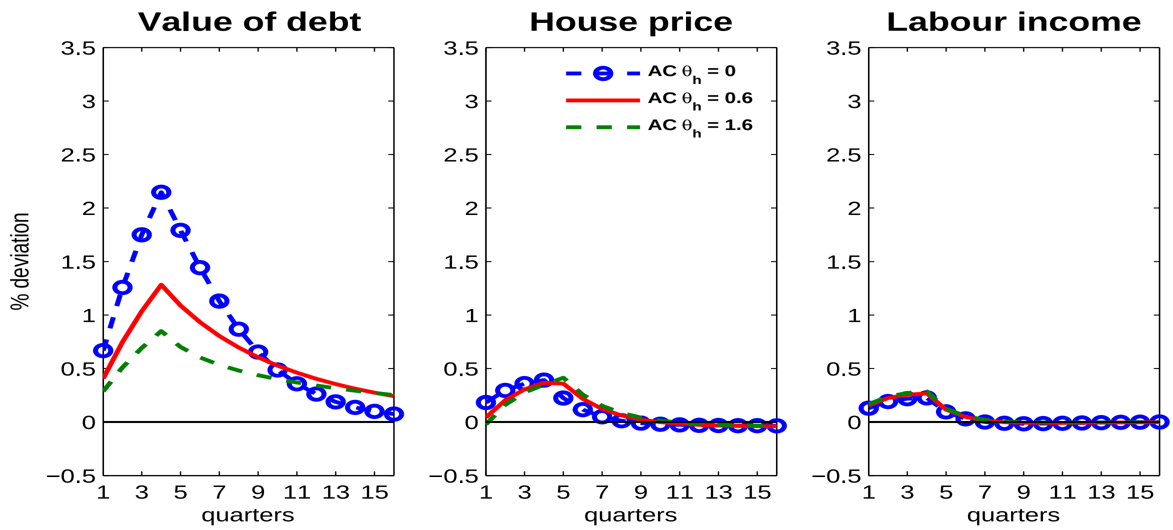


FIGURE 3.11: Mortgagor's value of debt ($S_t b_{t+1}$), house price (q_t^h) and real labor income ($w_t L_t$) responses to a 25bp decrease in the interest rate. **Different values for housing adjustment costs. The wage (Frisch) elasticity of labor supply is set to 2.**

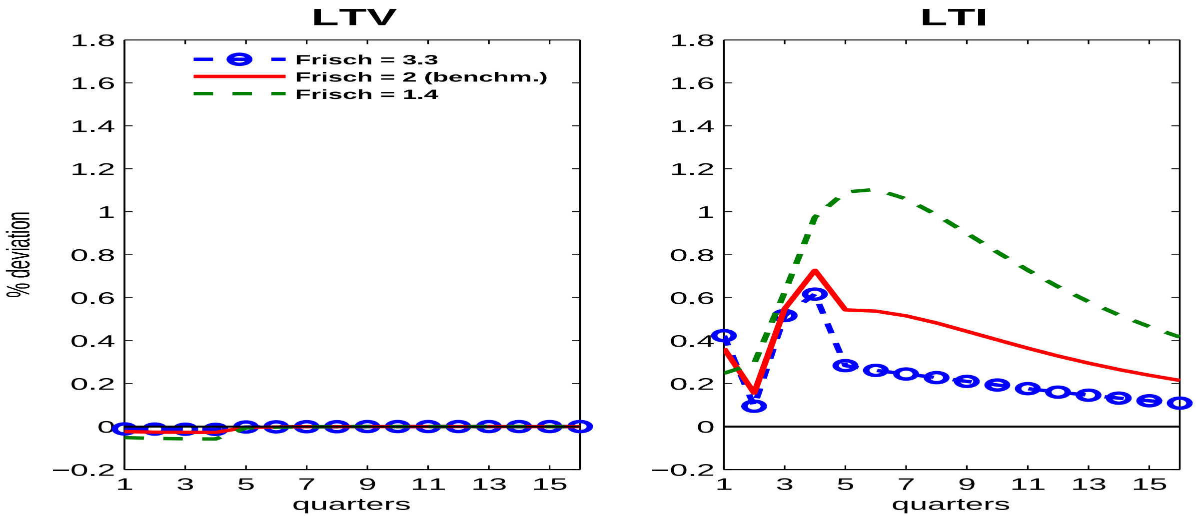


FIGURE 3.12: Mortgagor's LTV and LTI model responses to a 25bp decrease in the interest rate. **Different values for the wage (Frisch) elasticity of labor supply, computed in the model as $\frac{1}{\eta-1}$. Housing adjustment costs set to $\theta_h = 0.6$.** The LTV in the model is computed as $\frac{S_t b_{t+1}}{h_t q_t^h}$, while the LTI is computed as $\frac{S_t b_{t+1}}{L_t w_t}$.

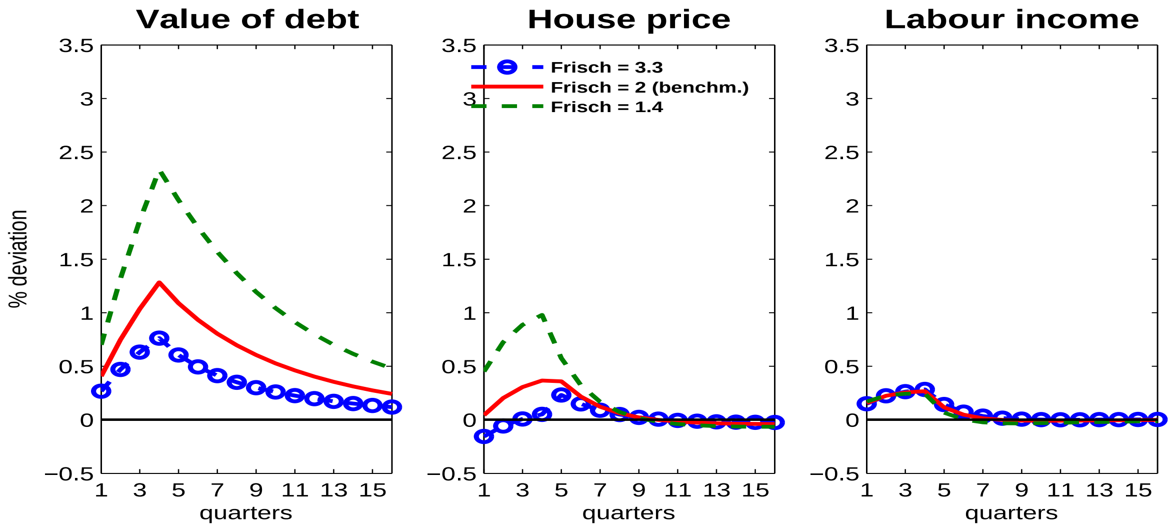


FIGURE 3.13: Mortgagor's value of debt ($S_t b_{t+1}$), house price (q_t^h) and real labor income ($w_t L_t$) responses to a 25bp decrease in the interest rate. **Different values for the wage (Frisch) elasticity of labor supply, computed in the model as $\frac{1}{\eta-1}$. Housing adjustment costs set to $\theta_h = 0.6$**

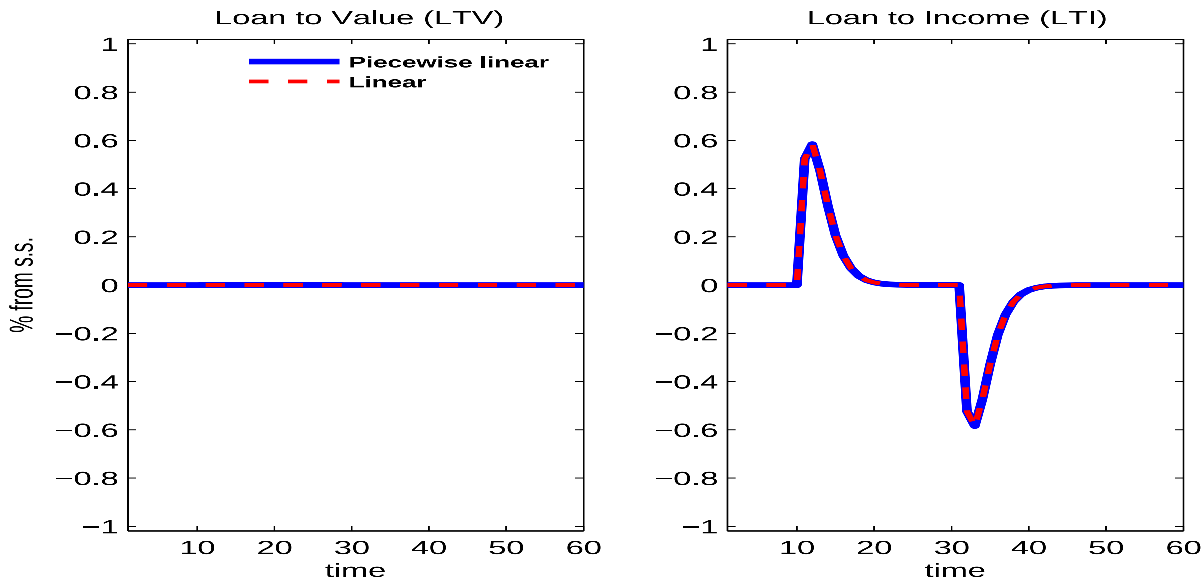


FIGURE 3.14: Mortgagor's LTV and LTI model responses to a **25bp decrease** and a subsequent **25bp unexpected increase** in the interest rate. **Benchmark parameter values as in table 3.4**. The LTV in the model is computed as $\frac{S_t b_{t+1}}{h_t q_t^h}$, while the LTI is computed as $\frac{S_t b_{t+1}}{L_t w_t}$.

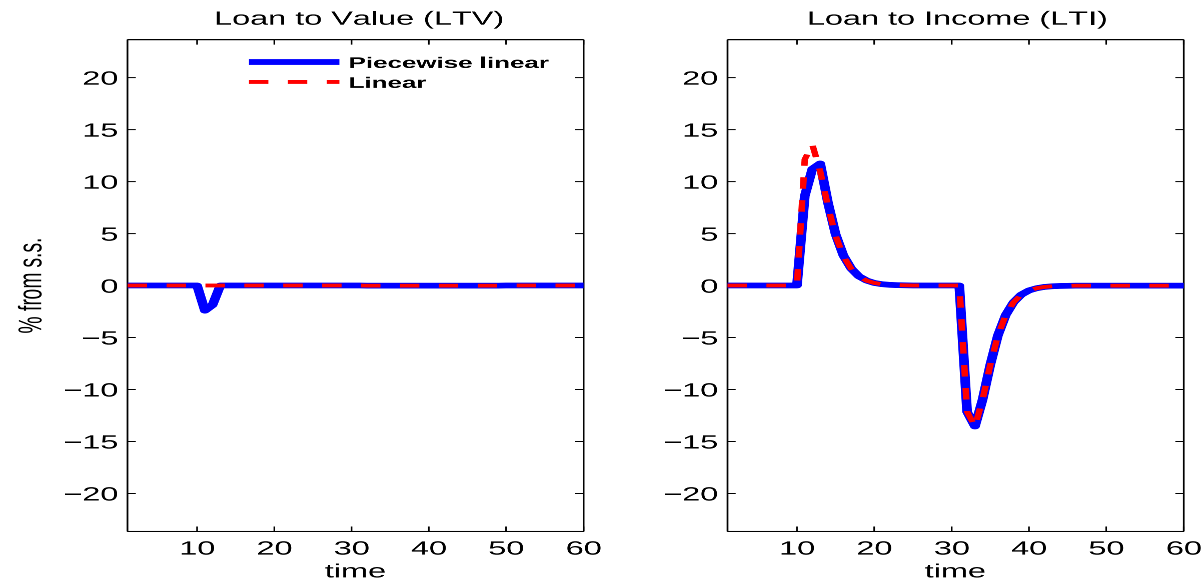


FIGURE 3.15: Mortgagor's LTV and LTI model responses to a **500bp unexpected decrease** and a subsequent **500bp unexpected increase** in the interest rate. **Benchmark parameter values as in table 3.4**. The LTV in the model is computed as $\frac{S_t b_{t+1}}{h_t q_t^h}$, while the LTI is computed as $\frac{S_t b_{t+1}}{L_t w_t}$.

Appendix A

Appendix Chapter 1

A.1 Feedback Systems in On-line Markets: Some Examples

The feedbacksystem is the backbone of online markets in which anonymous parties are transacting. Its role is to facilitate transactions by inducing trust in interactions in which traditional contractual agreements are not always enforceable. A feedback mechanism, otherwise called *reputation system*, is a public record of a trader's past transactions. Revealing an agent's past actions will affect the future behavior of the entire comunity towards that agent. It can hence discipline traders' behavior and provide them with incentives not to missbehave in the short-run.

In the early 2000, eBay was by far the most successful on-line market with almost fifty million users and twelve million auctions run daily. The success of the platform is due moslty to its feedback system. eBay's reputation system has a simple and for all users accessible design. At the end of each transaction, both parties can evaluate each other using ratings from a tripartite scale- positive (1), neutral (0) and negative (-1) - and an optional short comment. Following the argument in Cabral and Hortaçsu, (2010), according to which neutral grades are rather perceived as negative ratings, we devise our model with a binary rating system. We assume that traders in our model thuthfully report the outcome of each transaction as either a "Success"(S) or a "failure"(F). Under the assumption that bidders are interested in purchasing one object only and hence their aim is not to build a reputation, we restrict our attention to the sellers' track record. The aggregate measure used on eBay as a reputation indicator consists of the number of positive net of negative ratings. The bidders in our model are Bayesian updaters. The reputation measure they attach to a particular seller is their belief that the seller they are facing is of a certain type.

eBay's feedback system doesn't distinguish between feedbacks received from sellers or buyers. Cabral and Hortaçsu, (2010) documents that in general, traders start their career as buyers and accumulate a number of positive feedbacks. With an established reputation, they then start their seller's career. For simplicity, we assume that traders can be either sellers or buyers. Sellers in our model enter the game with a given reputation and build it gradually.

Even though eBay's feedback system promotes cooperation, it suffers from a couple of severe drawbacks. The free entry and exit for all market participants opens the door for opportunistic behavior. Sellers can choose to exit the market and re-enter under a new identity at basically no cost. Cabral and Hortaçsu, (2010) document that before exiting the market, sellers "milk" their reputation. In the benchmark version of our model we assume that sellers don't have the option to exit the market. In a further extension of our model we investigate the case in which sellers can exit the market once their reputation has fallen below a threshold and can opt for re-entry against paying a fixed cost.

A.2 Buyer's Selection Strategy and Sellers' Expected Revenues

Buyer's Selection Strategy

In what follows, we describe the main features that characterize the buyer's optimal selection strategy, and the resulting expected type of the second highest bidder in seller's j auction.
The *ex-ante* utility of a buyer with valuation v^i bidding in j's auction is defined as following:

$$\pi^i(v^i) = \sum_{n=1}^{N} Pr[n \text{ other bidders choose auction} j] \times Pr[\text{all } n \text{ other bidders have types} \leq v^i] \\ \times r_j \times (v^i - E[\text{2nd highest type in j's auction} \mid \text{highest type} = v^i \wedge \exists\, n \text{ other bidders}]) \tag{A.1}$$

Following Bapna, Dellarocas, and Rice, (2010), we introduce a series of measures that will prove useful in defining the equilibrium results. Given the auction selection strategy of a bidder $s : [0, 1] \times [0, 1]^J \Rightarrow [0, 1]^J$, the following function defines the probability that a buyer is of type v^i or lower *and* bids in seller's j auction.

$$Q_j(v^i|s) = \int_0^{v^i} s^i_j(u) f(u) du, \quad j = 1, \ldots, J$$

The expression $Q_j(1|s)$ stands for the probability that a random buyer bids in j's auction, whereas $1 - Q_j(1|s) + Q_j(v^i|s)$ is the probability that a random buyer is not of a higher type than v^i *and* bids in j's auction.
It results that the above defined expected profit of a buyer with valuation v^i bidding in j's auction can be written as: $\pi^i(v^i) = \int_0^{v^i} (1 - Q_j(1|s) + Q_j(x|s))^{N-1} dx$

Further we provide a short description of buyers' equilibrium strategies as outlined Proposition 3 in Dellarocas Bapna, Dellarocas, and Rice, (2010).

In the general set-up with J sellers in which we have assumed a complete order of their reputations, let there be an L defined as the lowest integer $2 \leq L \leq J - 1$ for which $r_{L+1} < \left(\frac{L-1}{\sum_{j=1}^{L} \sqrt[N-1]{\frac{1}{r_j}}}\right)^{N-1}$. If the integer does not exist, then $L = J$. The unique one-bid auction selection equilibrium is described as following:

1. Buyers are divided into L zones according to their types. Let t_z $z = 0, 1, \ldots, L$ with $t_0 = 1$, $t_L = 0$ be the zone delimiters. Buyers whose types satisfy $t_z < t \leq t_{z-1}$ belong to zone z.
2. Zone z buyers randomly chose among sellers $j = 1, \ldots, z$ with corresponding selection probabilities:
$$s_{zj} = \frac{\sqrt[N-1]{\frac{1}{r_j}}}{\sum_{j=1}^{z} \sqrt[N-1]{\frac{1}{r_j}}}$$
3. If $L < J$, then sellers $L+1, \ldots J$ are never chosen by any buyer
4. Zone delimiters t_z $z = 0, 1, \ldots, L-1$ are the solution to the following equations:
$$F(t_z) = \left(\sqrt[N-1]{r_{z+1}} \sum_{j=1}^{z} \sqrt[N-1]{\frac{1}{r_j}}\right) - (z-1)$$

Sellers' Expected Revenues

Lemma 3 in Dellarocas Bapna, Dellarocas, and Rice, (2010) provides the closed-form solution of seller's j expected revenue. In the auction of a seller with reputation r_j, where we assume w.l.o.g. $r_1 \geq \ldots \geq r_j \geq \ldots \geq r_M$, the expected type of the second highest bidder given their equilibrium strategies is given by:

$$H_{2,j}(r_1, ..., r_J) = 1 + (N-1) \int_0^1 (1 - Q_j(1) + Q_j(t))^N \, dt - N \int_0^1 (1 - Q_j(1) + Q_j(t))^{N-1} dt$$

The function $H_{2,j}(r_j, r_{-j})$ is increasing in seller's j reputation and decreasing in his competitors' reputations.

When reputations are equal $r_1 = r_2 = \ldots = r_J$, the expected type of the second highest bidder is the same in each auction and equals:

$$H_2(r_j, r_{-j}) = 1 + \frac{N-1}{J^N} \int_0^1 (M - 1 + F(t))^N dt - \frac{N}{M^{N-1}} \int_0^1 (M - 1 + F(t))^{N-1} dt$$

In the case of one seller in the market, the expected type of the second highest bidder is constant throughout the periods and equal to $H_2(N) = \frac{N-1}{N+1}$, assuming the monopolist faces the same number of buyers N every period whose valuations are drawn from the same distribution. In the case of only one seller or when all sellers have the same reputation, they profit vary linearly with their reputation.

Two Sellers

For the case of two sellers, further called i and j, our results are summarized in the following section. We assume that the reputations of the two sellers satisfy the following relation $r_{-j} > r_j$.

- *Assortative Matching:* there exists a threshold type $t_{j,-j}$ such that bidders with valuations higher than $t_{j,-j}$-bidders of zone $-j$- bid their expected valuation in the auction of the seller with the highest success probability, which we assume to be r_{-j}. Buyers whose types are below this threshold have a mixed optimal auction selection strategy that places a higher weight in choosing the auction of the seller with the lowest reputation r_j. Under the assumption of a uniform distribution of buyers' valuations, the threshold value and strategy selection vector take the following form:

$$\begin{aligned} t_{j,-j}(r_{-j}, r_j) = &\sqrt[N-1]{\frac{r_j}{r_{-j}}} \\ s_{-j} &= (s_{-j,-j}, s_{-j,j}) = (1, 0) \\ s_j &= (s_{j,-j}, s_{j,j}) = \left(\frac{\sqrt[N-1]{r_j}}{\sqrt[N-1]{r_j} + \sqrt[N-1]{r_{-j}}}, \frac{\sqrt[N-1]{r_{-j}}}{\sqrt[N-1]{r_j} + \sqrt[N-1]{r_{-j}}} \right) \end{aligned}$$

- As a result, the seller with the highest reputation, r_j, expects to get more bids and from buyers with higher valuations. The expected second highest valuation in his auction, henceforth called $H_{2,j}$ is given by

$$\begin{aligned} H_{2,-j}(r_{-j}, r_j) &= 1 + (N-1)\int_0^1 (1 - Q_{-j}(1) + Q_{-j}(t))^N \, dt - N\int_0^1 (1 - Q_{-j}(1) + Q_{-j}(t))^{N-1} dt \\ &= \frac{N-1}{N+1}\left(s_{j,-j}^N \left((1+t_{j,-j})^{N+1} - 1\right) + 1 - t_{j,-j}^{N+1}\right) - s_{j,-j}^{N-1}\left((1+t_{j,-j})^N - 1\right) + t_{j,-j}^N; \end{aligned}$$

- The seller with the relatively lower reputation, r_i, expects to get a lower number of bids than seller j and also from buyers with valuation from the low part of the distribution. The expected second highest type in i's auction is:

$$\begin{aligned} H_{2,j} &= 1 + (N-1)\int_0^1 (1 - Q_j(1) + Q_j(t))^N \, dt - N\int_0^1 (1 - Q_j(1) + Q_j(t))^{N-1} dt) \\ &= t_{j,-j} + \frac{N-1}{N+1}(1 - s_{j,-j})^N \left((1+t_{j,-j})^{N+1} - 1\right) - (1 - s_{j,-j})^{N-1}((1+t_{j,-j})^N - 1) \end{aligned}$$

A.3 Informative signals and the dynamics of beliefs

The way in which buyers (and other sellers) update their beliefs about the seller's type (trustworthiness) depends on the "informativeness" of the *signal* they receive. In our case, the signal is simply the realization of a transaction. Observing a success (S) or a failure (F) provides the buyer with information about the type of the seller (although both are unobservable, buyers and sellers *understand* the way in which other sellers behave *in equilibrium* and therefore they acknowledge the effort decisions by the different seller types. If the probability of an outcome, conditional on effort, is relatively "flat" in types, meaning that in equilibrium it does not change much with the type of a seller, then we say that the outcome is not an informative signal.

Along the previous lines, therefore, if we observe within the model that the updating of beliefs after a failure is much more gradual than after a success, this might be due to the characteristics of the conditional density of outcomes (conditional on effort): for certain parameter values, what might be happening is that the probability of a failure is relatively high (note that such probability is an equilibrium object which depends on the effort level). This implies that observing a success provides much more information about the type of the seller than observing a failure.

Lets see this with an example. In our model, we have that the probability of a success, conditional on effort and type is

$$p\left(r = s|e^{*}, \theta\right) = e \cdot \theta$$

Lets consider as an example the benchmark case, in which $\theta \in \{0.1, 0.9\}$. When we solve the model, for a particular cost function we obtain an optimal effort level $e^{*} \in [0.1, 0.15]$. This implies that the probability of a success, conditional on type is $p^{s}_{\theta=g} \in [0.09, 0.135]$ for the good type, and $p^{s}_{\theta=b} \in [0.01, 0.015]$ for the bad type.[1] The "likelihood ratio" *for a good type* is given by

$$LR_{\theta=g} \equiv \frac{p\left(r = s|\theta = g\right)}{p\left(r = s|\theta = b\right)} = 9$$

On the other hand, the conditional probabilities of a failure are $p^{f}_{\theta=g} \in [0.865, 0.91]$ and $p^{f}_{\theta=b} \in [0.985, 0.99]$ for good and bad types respectively. In this case, the likelihood ratio *for a bad type* is given by

$$LR_{\theta=b} \equiv \frac{p\left(r = f|\theta = b\right)}{p\left(r = f|\theta = g\right)} \in [1.088, 1.138]$$

The above implies that a success is much more informative about a good type than a failure about a bad type, and therefore gives rise to an asymmetry in the updating of beliefs.

[1] This is not entirely correct, since the optimal effort functions are different for good and bad types. However, with lower effort for bad types, the argument beow would be even more extreme.

A.4 Proofs

A.4.1 Proof of Lema 3

- **Separating Equilibria**

  In a separating equilibrium with commitment, a seller of type θ will announce e_1^θ in the first period and e_2^θ in the second period such that $e_1^g \neq e_1^b$ and $e_2^g \neq e_2^b$. In a separating equilibrium, upon the announcement of the policy any type would like to commit to, the type of the seller is immediately inferred and the reputation of the seller becomes degenerate.

  The problem for each type of seller would then be the following:

$$\max_{(e_1^\theta, e_2^\theta)\in[0,1]^2} \Pi^\theta = \theta e_1^\theta H_2 - c\left(e_1^\theta\right) + \delta\Big(\theta e_2^\theta H_2 - c\left(e_2^\theta\right)\Big) \tag{A.2}$$

  The FOCs are given by

$$\begin{aligned} c'\left(e_1^{\theta*}\right) &= \theta H_2 \\ c'\left(e_2^{\theta*}\right) &= \theta H_2 \end{aligned}$$

  implying $e_1^{\theta*} = e_2^{\theta*}$. However, unless the cost function is such that the profits of the two types are equal, $\Pi^g\left(e^{g*}\right) = \Pi^b\left(e^{b*}\right)$, one of the two types will have incentives to deviate and announce an effort level equal to the one from the other type. Thus, the separating equilibrium would break down.

- **Pooling equilibria**

  Pooling Equilibrium in the stage game

  We start our analysis by investigating the existence of a pooling equilibrium in the stage game. This correspond to analyzing the second period of our 2-period game.

  Definition 8 *A pooling equilibrium of the stage game is a Perfect Bayesian Equilibrium in which each type of agent chooses the same announcement in equilibrium:* $e_2^g = e_2^b = e_2^*$ *such that* $Prob(\theta = g|e_2^*) = \mu$ *and* $Prob(\theta = b|e_2^*) = 1-\mu$ *and the expected amount bid by a principal with valuation* v^i *is* $b(v^i) = v^i(\mu g + (1-\mu)b)e_2^*$

  The expected equilibrium stage profits of a monopolist in a pooling equilibrium are given by the following expression:

$$\pi(e_2^*) = (\mu g + (1-\mu)b)e_2^* H(N) - c(e_2^*) \tag{A.3}$$

  where $H(N) = \frac{N-1}{N+1}$ is the expected second highest bid in the auction of the monopolist. To characterize the set of pooling equilibria, we need to specify first how beliefs are formed out of equilibrium. Suppose now that, whenever a seller deviates from e_2^*, buyers will attach probability 1 of him being a bad type, i.e. $\mu = 0$. Whenever a seller deviates from e_2^* and chooses to announce $\tilde{e}_2$ instead, his expected deviation profits are described by:

$$\pi(\tilde{e}_2) = b\tilde{e}_2 H(N) - c(\tilde{e}_2) \tag{A.4}$$

Further we compute a monopolist's maximum attainable profits under deviation. The effort level that maximizes the deviation profits of the monopolist can be computed from the FOC:

$$c'(\tilde{e}_2^*) = bH(N) \tag{A.5}$$

and equals:

$$\tilde{e}_2^* = c'^{-1}(bH(N)) \tag{A.6}$$

The maximum attainable profit under deviation is given by $\pi(\tilde{e}_2^*)$. Further, we assume that when deviating, the seller will choose $\tilde{e}_2^*$.
We investigate now what efforts can be sustained in a pooling equilibrium. Incentive compatibility requires that $\pi(e_2^*) \geq \pi(\tilde{e}_2^*)$. The set of pooling equilibria, as illustrated in figure A.1, is composed of all the effort levels comprised in the interval $[\underline{\mathrm{e}}, e_2^{*d}]$, where e_2^{*d} is the effort level that makes the incentive constraint hold with equality $\pi(e_2^{*d}) = \pi(\tilde{e}_2^*)$. In figure A.1, e_2^{*d} is the value on the x-Axis corresponding to the crossing of *Equilibrium Profits* line with the *Maximum Deviation Profits*. Hence, the set of pooling equilibrium is given by:

$$\begin{aligned} S_p = \{(e_2^{*g}, e_2^{*b}) | e_2^{*g} = e_2^{*b} = e_2^*, \text{with } e_2^* \in [\underline{\mathrm{e}}, e_2^{*d}], \\ \text{where } e_2^{*d} \text{ is the solution to } \pi(e_2^{*d}) = \pi(\tilde{e}_2^*), \tilde{e}_2^* = c'^{-1}(bH(N))\} \end{aligned}$$

Solving for e_2^{*d} depends on the functional form of the cost function. For our choice of cost function, this amounts to solving a third degree equation.

Pooling Equilibrium in the repeated game

We now go back to the first period and analyze the possibility of having a pooling equilibrium, keeping in mind the effect of announced efforts on the updating of beliefs and on the induced probability distribution over future outcomes.

The question we address at this point is, whether, given the set of pooling equilibrium of the second period, can pooling be sustained in the first period as well? The analysis follows a similar pattern as the one above.

Suppose there exist a pooling equilibrium, in which both seller types announce $e_1^g = e_1^b = e_1^*$ at the beginning of the first period, given . As in the last stage,

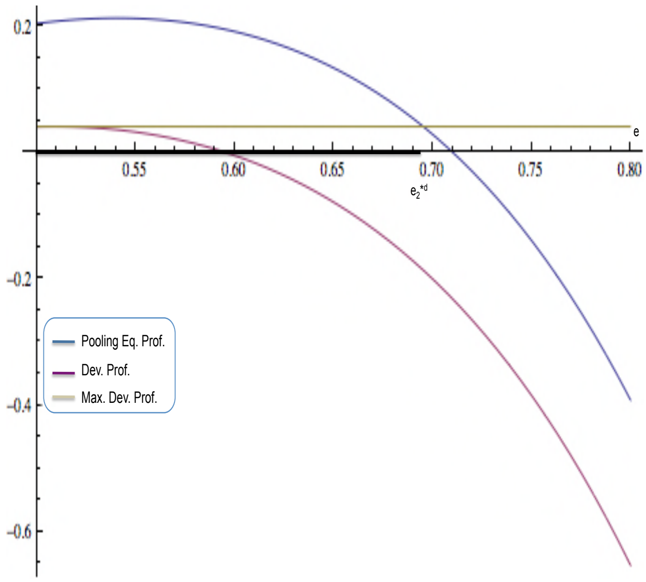


FIGURE A.1: One Period Pooling Equilibrium

the off-equilibrium-path beliefs assign a zero probability to the seller being of good type,when deviating from e_1^*. Remark that, a deviation in the first period will attribute a "bad seller" label to the deviating seller both in the first *and* in the second period.

For simplicity, assume that the equilibrium effort chosen in the second period is the same, no matter the outcome of the first period. The equilibrium inter-temporal profit of a seller of type θ, in which the efforts are pooled in both periods can be written as:

$$\begin{aligned}
\Pi(e_1^*, e_2^*, \theta) =& \pi_1^* + \delta(\theta e_1^* \pi_2^{*S} + (1-\theta e_1^*)\pi_2^{*F}) \text{ where} \\
\pi_1^* &= (\mu g + (1-\mu)b)e_1^* H(N) \\
\pi_2^{*S} &= (\mu^S g + (1-\mu^S)b)e_2^* H(N) \\
\pi_2^{*F} &= (\mu^F g + (1-\mu^F)b)e_2^* H(N)
\end{aligned}$$

On the equilibrium path, beliefs update according to Bayes rule.

$$\begin{aligned}
\mu^S &= \frac{\mu g}{\mu g + (1-\mu)b} \\
\mu^F &= \frac{\mu(1-ge_1^*)}{\mu(1-ge_1^*) + (1-\mu)(1-be_1^*)}
\end{aligned}$$

When the seller deviates by announcing $\tilde{e_1^*}$ in the first period, the buyers will infer that he is of bad type. This information will be transmitted to the next generation of buyers. Hence, after deviating in the first period, the best choice in the second period is to play $\tilde{e_2^*}$ in the second period. Hence, the inter temporal profits from deviating in the first period are:

$$\begin{aligned}\Pi^d(\tilde{e^*}_1, \tilde{e^*}_2, \theta) =& \pi_1^{*d} + \delta\pi_2^{*d} \text{ where}\\ & \pi_t^{*d} = bH(N)\tilde{e^*}_t, \ \ t = 1, 2\end{aligned}$$

The maximum attainable profit at deviation, is obtained when employing an effort level $\tilde{e^*}_t = c'^{-1}(bH(N))$. This effort is optimal in both periods, as beliefs don't play any role in this case (for these specified out-of-eq beliefs).

$$\begin{aligned}\Pi^d(\tilde{e^*}_1, \tilde{e^*}_2, \theta) &= (1+\delta)\pi^{*d}\\ &= (1+\delta)(bH(N)c'^{-1}(bH(N)))\end{aligned}$$

We turn back now to characterize the set of pooling equilibrium of the first period.

First, observe that inter temporal profits for the bad type are always lower than those of the good type, i.e. $\min(\Pi(e_1^*, e_2^*, g), \Pi(e_1^*, e_2^*, b)) = \Pi(e_1^*, e_2^*, b)$.

To determine the set of pooling equilibria, we must make sure the announced choice is incentive compatible for both types. Given the above relation between types' profits, the only constraint that needs to hold is:
$\Pi(e_1^*, e_2^*, b) \geq \Pi^d(\tilde{e^*}_1, \tilde{e^*}_2, \theta)$

In the first panel of figure A.2, we observe how the intertemporal profits of the bad type (blue area) relate to a deviating seller's profits (green area). For values of e_1^* and e_2^* for which the blue surface is above the green surface, we have that deviation from the pooling equilibrium never pays. The lower figure provides a better picture of the possible pairs of effort levels that can be sustained in a pooling equilibrium. A formal description of this set is given by:

$$\begin{aligned}S_{pI} = \{((e_1^{*g}, e_1^{*b}), (e_2^{*g}, e_2^{*b}))|&e_2^{*g} = e_2^{*b} = e_2^* \wedge e_1^{*g} = e_1^{*b} = e_1^* \text{such that}\\ &(e_1^*, e_2^*) \in [\underline{\mathrm{e}}, e_2^{*d}] \times [\underline{\mathrm{e}}, e_1^{*d}] \text{where } e_2^{*d} \text{and} e_2^{*d}\\ &\text{are jointly determined from the relation } \ \Pi(e_1^{*d}, e_2^{*d}) = \pi(\tilde{e}_1^*, \tilde{e}_2^*), \}\end{aligned}$$

The points (e_1^{*d}, e_2^{*d}) describe precisely the border between the blue and the green surface in the right panel of figure A.2.

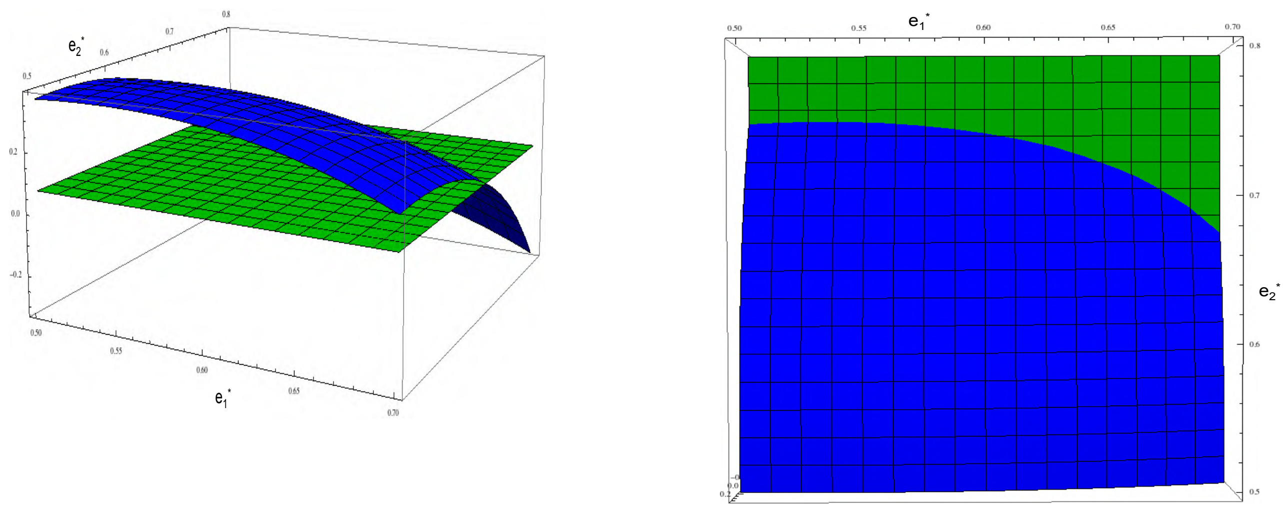


FIGURE A.2: Pooling Eq in the intertemporal game

A.4.2 Proposition 1

An equilibrium of the two-periods game where a monopolist of unobservable type chooses higher than minimum effort exists and is unique.

A.4.3 Proof of Proposition 1

We start by proving that a pooling equilibrium cannot exist.

First, we notice that both types exerting minimal effort level cannot be a solution since the marginal cost of effort is lower than the marginal benefit at this level. Furthermore, given the monotonicity and strict convexity of the cost function and given the linear delivery technology, it follows that a given effort level for both types, the good type has a higher marginal benefit compared to the bad type. In equilibrium the good type will employ more effort than the bad type.

The solution to the monopolist's problem must satisfy following FOCs:

$$\begin{cases} c'(e_1^{*g}) = & gf(e_1^{*g}, e_1^{*b}) \\ c'(e_1^{*b}) = & bf(e_1^{*g}, e_1^{*b}) \end{cases} \tag{A.7}$$

where $f(e^{*g}, e^{*b}) = \delta H_2^m \underline{e} \left(\mu^S(e_1^{*g}, e_1^{*b}) - \mu^F(e_1^{*g}, e_1^{*b})\right)$ and $H_2^m = \frac{N-1}{N+1}$ is the expected second highest valuation in the monopolist's auction. e_1^{*g}, e_1^{*b} are the equilibrium effort levels of the good type resp. the bad type monopolist in period 1.

First, we prove that $f_{e^g}(e^g, e^b) > 0$ and $f_{e^b}(e^g, e^b) < 0$.

Recall that $\mu^S = \frac{\mu g e^g}{\mu g e^g + (1-\mu) b e^b}$ and $\mu^F = \frac{\mu(1-ge^g)}{\mu(1-ge^g)+(1-\mu)(1-be^b)}$.

It easily follows that $\frac{\partial \mu^S}{\partial e^g} = \frac{\mu(1-\mu)gbe^b}{(\mu g e^g+(1-\mu)be^b)^2} > 0$ and $\frac{\partial \mu^F}{\partial e^g} = \frac{-\mu(1-\mu)gbe^b}{(\mu(1-ge^g)+(1-\mu)(1-be^b))^2} < 0$. Hence:

$$f_{e^g}(e^g, e^b) = \underbrace{\frac{\partial \mu^S}{\partial e^g}}_{>0} - \underbrace{\frac{\partial \mu^F}{\partial e^g}}_{<0} > 0 \tag{A.8}$$

Similar analysis applies for $f_{e^b}(e^g, e^b)$: $\frac{\partial \mu^S}{\partial e^b} = \frac{-\mu(1-\mu)gbe^g}{(\mu g e^g+(1-\mu)be^b)^2} < 0$ and $\frac{\partial \mu^F}{\partial e^b} = \frac{\mu(1-\mu)gbe^g}{\mu(1-ge^g)+(1-\mu)(1-be^b))^2} > 0$.
Hence:

$$f_{e^b}(e^g, e^b) = \underbrace{\frac{\partial \mu^S}{\partial e^b}}_{<0} - \underbrace{\frac{\partial \mu^F}{\partial e^b}}_{>0} < 0 \tag{A.9}$$

We are looking for an interior solution. Since $c'(\underline{e}) = 0$ and $f(e^g, \underline{e}) > 0$ and resp. $f(\underline{e}, e^b) > 0$, it follows that $c'(\underline{e}) - gf(\underline{e}, e^b) < 0$ and $c'(\underline{e}) - bf(e^g, \underline{e}) < 0$ for $e^g < e^{*g}$and $e^b < e^{*b}$ respectively. In equilibrium A.7 holds, namely $c'(e^{*b}) - bf(e^{*g}, e^{*b}) = 0$ and $c'(e^{*g}) - gf(e^{*g}, e^{*b}) = 0$.
It follows that on the $[\underline{e}, e^{*b}] \times [\underline{e}, e^{*g}]$: $c'(e^b) - bf(e^g, e^b)$ is increasing in e^b and $c'(e^g) - gf(e^g, e^b)$ is increasing in e^g.

$$\begin{aligned} \frac{\partial \left(c'(e^g) - gf(e^g, e^b)\right)}{\partial e^g} |_{e^{*g}} > 0 \\ \frac{\partial \left(c'(e^b) - bf(e^g, e^b)\right)}{\partial e^b} |_{e^{*b}} > 0 \end{aligned} \tag{A.10}$$

Hence:

$$\begin{aligned} c''(e^{*g}) > g f_{e^g}(e^{*g}, e^b) \\ c''(e^{*b}) > b f_{e^b}(e^g, e^{*b}) \end{aligned} \tag{A.11}$$

$\forall\ e^g \in [\underline{e}, e^{*g}]$, $e^b \in [\underline{e}, e^{*b}]$.

We take implicit derivatives of the FOCs in A.7

$$\begin{cases} c''(e^{*g})\frac{\partial e^{*g}}{\partial e^b} = & gf_{e^{*g}}(e^{*g}, e^b)\frac{\partial e^{*g}}{\partial e^b}|_{(e^{*g}, e^{*b})} + gf_{e^b}(e^{*g}, e^{*b}) \\ c''(e^{*b})\frac{\partial e^{*b}}{\partial e^g} = & bf_{e^{*g}}(e^g, e^{*b}) + bf_{e^b}(e^{*g}, e^{*b})\frac{\partial e^{*b}}{\partial e^g}|_{(e^{*g}, e^{*b})} \end{cases} \Rightarrow \tag{A.12}$$

$$\Rightarrow \begin{cases} \left(c''(e^{*g}) - gf_{e^g}(e^{*g}, e^{*b})\right)\frac{\partial e^{*g}(e^b)}{\partial e^b}|_{(e^{*g}, e^{*b})} = & gf_{e^b}(e^{*g}, e^{*b}) > 0 \\ \left(c''(e^{*b}) - bf_{e^b}(e^{*g}, e^{*b})\right)\frac{\partial e^{*b}(e^g)}{\partial e^g}|_{(e^{*g}, e^{*b})} = & bf_{e^g}(e^{*g}, e^{*b}) < 0 \end{cases} \tag{A.13}$$

From A.8, A.9 and A.11 it follows that $\frac{\partial e^{*g}}{\partial e^b}|_{(e^{*g}, e^{*b})} < 0$ and $\frac{\partial e^{*b}}{\partial e^g}|_{(e^{*g}, e^{*b})} > 0$. Hence best responses are monotonically decreasing respectively increasing. Hence, there exists just one intersection of the best responses, which implies that the equilibrium is unique. ■

A.4.4 Proposition 2

For any type θ, $\theta \in \{b, g\}$, in the equilibrium with hidden effort and unobserved type, the equilibrium effort level:

1. increases with the discount factor δ
2. The effort level of the good type is increasing in g and decreasing in b. The opposite is true for the effort level of the bad type.
3. is a concave in initial beliefs μ_0
4. For any $g, b, \delta \in [0, 1]$ it holds that $e_1^{\theta obsNC} < e_1^{*\theta} \leq e_1^{\theta obsC} < e_1^{\theta eff}$. Moreover,

A.4.5 Proof of Proposition 2

The first order conditions are given by:

$$c'(e_1^{*\theta}) = \delta\theta H_2^m(\mu_1^S - \mu_1^F)(g - b)\underline{e}$$

1. *For any type, the effort level is increasing in the discount factor* δ

   We take the derivative of the F.O.C. with respect to δ keeping in mind that the equilibrium effort $e_1^{*\theta}$ depends on δ as well:

$$\underbrace{c''(e_1^{*\theta})}_{>0}\frac{\partial e_1^{*\theta}}{\partial \delta} = \underbrace{\theta H_2^m(\mu_1^S - \mu_1^F)(g - b)\underline{e}}_{>0}$$

$$\Leftrightarrow \frac{\partial e_1^{*\theta}}{\partial \delta} > 0$$

2. *For any type, the effort level of the good type is increasing in g and decreasing in b. The opposite is true for the effort level of the bad type.*

   We look first at the first derivatives of μ_1^S and μ_1^F w.r.t g and b.

$$\frac{\partial \mu_1^S}{\partial g} = \frac{\mu b e^b e^g (1-\mu)}{b e^b (1-\mu) + e^g g \mu)^2} > 0$$
$$\frac{\partial \mu_1^S}{\partial b} = -\frac{\mu g e^b e^g (1-\mu)}{b e^b (1-\mu) + e^g g \mu)^2} < 0$$
$$\frac{\partial \mu_1^F}{\partial g} = -\frac{\mu (1 - b e^b) e^b e^g (1-\mu)}{(1 - (b e^b (1-\mu) + e^g g \mu))^2} < 0$$
$$\frac{\partial \mu_1^F}{\partial b} = \frac{\mu (1 - g e^g) e^b (1-\mu)}{(1 - (b e^b (1-\mu) + e^g g \mu))^2} > 0$$

   It easily follows that:

$$\underbrace{c''(e_1^{*\theta})}_{>0} \frac{\partial e_1^{*\theta}}{\partial g} = \delta H_2^1 (\mu_1^S - \mu_1^F) e^{*\theta obs} + \delta g H_2^1 e^{*\theta obs} \underbrace{\underbrace{\frac{\partial \mu_1^S}{\partial g}}_{>0} - \underbrace{\frac{\partial \mu_1^S}{\partial b}}_{<0}}_{>0}$$
$$\Leftrightarrow \frac{\partial e_1^{*\theta}}{\partial g} > 0$$

   and

$$\underbrace{c''(e_1^{*\theta})}_{>0} \frac{\partial e_1^{*\theta}}{\partial b} = \delta H_2^1 (\mu_1^S - \mu_1^F) e^{*\theta obs} + \delta g H_2^1 e^{*\theta obs} \underbrace{\underbrace{\frac{\partial \mu_1^F}{\partial b}}_{<0} - \underbrace{\frac{\partial \mu_1^F}{\partial b}}_{>0}}_{<0}$$
$$\Leftrightarrow \frac{\partial e_1^{*\theta}}{\partial b} < 0$$

3. *For any type, the effort level is first increasing then decreasing in μ_0 (effort level is a concave function of μ_0)*

   (a) We notice first that when $\mu_0 = 0$ or $\mu_0 = 1 \Rightarrow \mu_1^S = \mu_1^F = 0$ respectively $\mu_1^S = \mu_1^F = 1$. This implies in both cases that the RHS of the F.O.Cs is 0. $\Rightarrow c'(e_1^{*\theta}) = 0$. Hence, the optimal effort level when initial beliefs are either 0 or 1 is the maximal costless effort, i.e. $e_1^{*\theta} = e^{*\theta obs}$

(b) For $\mu_0 > 0$, derive the F.O.Cs implicitly w.r.t. μ_0.

$$\underbrace{c''(e_1^{*\theta})}_{>0}\frac{\partial e_1^{*\theta}}{\partial \mu_0} = \delta\theta H_2^1 \frac{\partial(\mu_1^S - \mu_1^F)}{\partial \mu_0} e_2^{*\theta obs}$$

$$\frac{\partial(\mu_1^S - \mu_1^F)}{\partial \mu_0} = \frac{ge^{*g}be^{*b}}{(\mu_0 ge^{*g} + (1-\mu_0)be^{*b})^2} - \frac{(1-ge^{*g})(1-be^{*b})}{(1-(\mu_0 ge^{*g} + (1-\mu_0)be^{*b}))^2} \tag{A.14}$$

$$\frac{\partial(\mu_1^S - \mu_1^F)}{\partial \mu_0}|_{\mu_0=0} = \underbrace{\frac{ge^{*g}}{be^{*b}}}_{>1} - \underbrace{\frac{(1-ge^{*g})}{(1-be^{*b})}}_{<1} > 0 \quad \text{and}$$

$$\frac{\partial(\mu_1^S - \mu_1^F)}{\partial \mu_0}|_{\mu_0=1} = \underbrace{\frac{be^{*b}}{ge^{*g}}}_{<1} - \underbrace{\frac{(1-be^{*b})}{(1-ge^{*g})}}_{>1} \quad \text{and} \tag{A.15}$$

In the first expression A.15 we state the derivative of the RHS of the F.O.Cs w.r.t. μ_0. The denominator of the first fraction is strictly increasing in μ_0, which implies that the first fraction is decreasing in μ_0. The opposite is true for the second fraction. It follows immediately that $\frac{\partial(\mu_1^S - \mu_1^F)}{\partial \mu_0}$ A.15 is strictly decreasing in μ_0. It must be then the case that up to a threshold value of μ_0, the RHS of the F.O.Cs is positive, which implies that $\frac{\partial e_1^{*\theta}}{\partial \mu_0} > 0$. For values of μ_0 which are higher than the threshold, the RHS of the F.O.Cs turns negative, which implies $\frac{\partial e_1^{*\theta}}{\partial \mu_0} < 0$. Hence the optimal effort level is concave in μ_0.

A.4.6 Proof of Proposition 3

Numerical.

Lemma 4

Single-Crossing condition: Let $e_{j,t} : \Theta_j \times \mathcal{H}^t \longrightarrow [0,1]$ be a decision rule for seller $j = 1, 2$, as a function of her type $\theta_j \in \Theta_j$ and a history $h^t \in \mathcal{H}^t$. Then, given the multiplicative technology $P(y_j = S|\theta, e) = \theta e$, it is the case in equilibrium that $e_j(g; h^t) \equiv e_j^g(h^t) \geq e_j^b(h^t)$, $\forall h^t \in \mathcal{H}^t$, $\forall j$.

A.4.7 Proof of Lema 4

Define $e^g \in [0,1]$ and $e^b \in [0,1]$ as the equilibrium effort decisions for a seller of type g and type b respectively. Relying on a revealed preference argument, it must then be the case that, for seller j of type θ

$$\Pi_{j,0}\left(e^{\theta}_{j,1};\theta\right) \geq \Pi_{j,0}\left(e^{\theta'}_{j,1};\theta\right)$$

This implies, for type g and b,

$$c\left(e^{g}\right) - c\left(e^{b}\right) \leq \delta g\left(e^{g} - e^{b}\right)\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + \left(1 - r_{-j,1}\right)\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right] \tag{A.16}$$

and

$$c\left(e^{g}\right) - c\left(e^{b}\right) \geq \delta b\left(e^{g} - e^{b}\right)\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + \left(1 - r_{-j,1}\right)\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right] \tag{A.17}$$

Here,

$$\pi^{y_{1,1},y_{2,1}}_{j,2} = \underline{e}\left(g\mu^{y_{j,1}}_{j,1} + b\left(1 - \mu^{y_{j,1}}_{j,1}\right)\right)H_{2,j}\left(r_{1,2}, r_{2,2}\right) - c\left(\underline{e}\right)$$

is the profit for seller j in period 2 after the outcomes $\left(y_{j,1}, y_{-j,1}\right)$ have been realized in period 1, and

$$r_{-j,1} = g e^{g}_{-j,1}\mu_{-j,0} + b e^{b}_{-j,1}\left(1 - \mu_{-j,0}\right)$$

is the *reputation* of seller $-j$ in period 1, with initial type-beliefs given by $\left(\mu_{j,0}, \mu_{-j,0}\right)$.

Summing eq.(A.16) and eq.(A.17) we have

$$\delta\left(e^{g} - e^{b}\right)\left(g - b\right) \geq 0$$

which, given the assumption $0 < b < g \leq 1$, implies that $e^{g} \geq e^{b} > \underline{e}$. ■

A.4.8 Proof of Proposition 4

Existence

The equilibrium is described by the following system of equations:

$$\begin{cases} c'\left(e^{*g}_{j,1}\right) & = g\delta\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + \left(1 - r_{-j,1}\right)\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right] \\ c'\left(e^{*b}_{j,1}\right) & = b\delta\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + \left(1 - r_{-j,1}\right)\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right] \\ c'\left(e^{*g}_{-j,1}\right) & = g\delta\left[r_{j,1}\left(\pi^{SS}_{-j,2} - \pi^{FS}_{-j,2}\right) + \left(1 - r_{j,1}\right)\left(\pi^{SF}_{-j,2} - \pi^{FF}_{-j,2}\right)\right] \\ c'\left(e^{*b}_{-j,1}\right) & = b\delta\left[r_{j,1}\left(\pi^{SS}_{-j,2} - \pi^{FS}_{-j,2}\right) + \left(1 - r_{j,1}\right)\left(\pi^{SF}_{-j,2} - \pi^{FF}_{-j,2}\right)\right] \end{cases} \tag{A.18}$$

where $r_{j,1}$ is the perceived probability of success of seller j at time 1, $\pi^{y_j,y_{-j}}_{j,2}$ is the time 2 profit of seller j given that the two players have registered the outcomes $y_j, y_{-j} \in \{S, F\}$

To prove the existence of a MPE of the two-stage game with two sellers we use Brouwer's fixed point theorem.

We start first by characterizing the marginal cost function $c'(\cdot)$ and its inverse $c'^{-1}(\cdot)$. $c' : [\underline{e}, 1) \to [0, \infty)$ is a monotonically increasing and strictly convex function. Hence $c'(\cdot)$ is a bijection. This guarantees the existence of well defined inverse function $c'^{-1} : [0, \infty) \to [\underline{e}, 1)$.

The above system of FOC's hence translates into:

$$\begin{cases} e^{*g}_{j,1} & = c'^{-1}\left(g\delta\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + (1 - r_{-j,1})\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right]\right) \\ e^{*b}_{j,1} & = c'^{-1}\left(b\delta\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + (1 - r_{-j,1})\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right]\right) \\ e^{*g}_{-j,1} & = c'^{-1}\left(g\delta\left[r_{j,1}\left(\pi^{SS}_{-j,2} - \pi^{FS}_{-j,2}\right) + (1 - r_{j,1})\left(\pi^{SF}_{-j,2} - \pi^{FF}_{-j,2}\right)\right]\right) \\ e^{*b}_{-j,1} & = c'^{-1}\left(b\delta\left[r_{j,1}\left(\pi^{SS}_{-j,2} - \pi^{FS}_{-j,2}\right) + (1 - r_{j,1})\left(\pi^{SF}_{-j,2} - \pi^{FF}_{-j,2}\right)\right]\right) \end{cases} \tag{A.19}$$

Denote now $\boldsymbol{e} \equiv \left(e^g_j, e^b_j, e^g_{-j}, e^b_{-j}\right)$ the 4-dimensional vector of efforts (actions) for seller-types, with $\boldsymbol{e} \in \mathcal{E}^4 = [\underline{e}, \bar{e}]^4$. Note that $\mathcal{E}$ is a convex and compact subset of the one-dimensional Euclidean space.

For $\boldsymbol{e} > 0$, the type-belief μ, the success probability function $r(\mu_0, e) \equiv P(y_j = S)$ and the expected valuation of the second highest type in seller's j auction, $H_{j,2}(\boldsymbol{r})$, are all vector-valued functions *continuous* in $\boldsymbol{e}$.

Define the function (for a fixed δ, θ and μ_0) $S^\theta_j : \mathcal{E}^4 \longrightarrow D$ with $D \subset \mathbb{R}$ and $i = 1, 2$ as

$$S^\theta_j(\boldsymbol{e}; \theta, \delta, \mu_0) = \theta\delta\left[r_{-j,1}\left(\pi^{SS}_{j,2} - \pi^{FS}_{j,2}\right) + (1 - r_{-j,1})\left(\pi^{SF}_{j,2} - \pi^{FF}_{j,2}\right)\right]$$

where $\pi^{y_j, y_{-j}}_{j,2} = r^{y_j}_j H_{j,2}$ is the profit for seller j in period 2 after the (joint) outcome for seller j and $-j$ $\boldsymbol{y} = (y_j, y_{-j})$. One can readily see that S^θ_j is a continuous and bounded function.

Hence, also the function composition $\left(c'^{-1} \circ S^\theta_j\right) : \mathcal{E}^4 \longrightarrow \mathcal{E}$ is continuous and bounded. Denote the vector function composition as $(c'^{-1} \circ \boldsymbol{S}) : \mathcal{E}^4 \longrightarrow \mathcal{E}^4$.

So far we have proven that $\mathcal{E}$ is a convex and compact subset of the 4-dimensional euclidean space on which a continuous and bounded function $(c'^{-1} \circ \boldsymbol{S})$ is defined.

Then, Brouwer's fixed point theorem guarantees the existence of a fix point: $\exists \boldsymbol{e}^*$ s.t. $\boldsymbol{e}^* = (c'^{-1} \circ \boldsymbol{S})(\boldsymbol{e}^*)$.

Uniqueness

To prove uniqueness of this equilibrium, we use the results developed by Mason and Valentinyi Mason and Valentinyi, (2007). Their paper provides sufficient conditions for existence and uniqueness of equilibrium in monotone pure strategies in a broad class of Bayesian games. When incremental interim payoffs satisfy the *uniform single crossing* and the *Lipschitz continuity* (with respect to other's strategy) condition, then a contraction mapping argument can be applied to prove the existence and uniqueness of the equilibrium.

In our setup, the interim payoff of a seller j (when seller j knows its type θ_j, $\theta_j \in \{g, b\}$) is given by its expected inter-temporal profit.

$$\Pi_j = r_{j,1}H_{2,j}(r_{j,1}, r_{-j,1}) - c(e^{\theta_j}_{j,1}) + \delta[\theta_j e^{\theta_j}_{j,1} r_{-j,1}(r^S_{j,1} H_{2,j}(r^S_{j,2}, r^S_{-j,2})) + \theta e^{\theta_j}_{j,1}(1 - r_{-j,1})(r^S_{j,1} H_{2,j}(r^S_{j,2}, r^F_{-j,2}))) \\ (1 - \theta_j e^{\theta_j}_{j,1}) r_{-j,1}(r^F_{j,1} H_{2,j}(r^F_{j,2}, r^S_{-j,2})) + (1 - \theta_j e^{\theta_j}_{j,1})(1 - r_{-j,1})(r^F_{j,1} H_{2,j}(r^F_{j,2}, r^F_{-j,2}))] \tag{A.20}$$

Bearing in mind that $r_{j,1} = \mu_j g\tilde{e}^g_{j,1} + (1-\mu_j) b\tilde{e}^b_{j,1}$, and denoting period 1 payoff by $\boldsymbol{R_{j,1}} = r_{j,1}H_{2,j}(r_{j,1}, r_{-j,1})$ and period 2 payoffs respectively $\boldsymbol{R_{j,2}}(y_j, y_{-j}) = r^{y_j}_{j,2} H_{2,j}(r^{y_j}_{j,2}, r^{y_{-j}}_{-j,2})$ where $y_j, y_{-j} \in \{S, F\}$ represent the outcomes of the transactions between period 1 and period 2, we can rewrite the expression in the following way:

$$\begin{aligned} \Pi_j = \mu_{-j,1}\Bigg[&\boldsymbol{R_{j,1}} - c(e^{\theta_j}_{j,1}) + \delta\Bigg[g\tilde{e}^g_{-j,1}\Bigg(\theta_j e^{\theta_j}_{j,1}\boldsymbol{R_{j,2}}(S,S) + (1-\theta_j e^{\theta_j}_{j,1})\boldsymbol{R_{j,2}}(F,S)\Bigg) + \\ &+ (1 - g\tilde{e}^g_{-j})\Bigg(\theta_j e^{\theta_j}_{j,1}\boldsymbol{R_{j,2}}(S,F) + (1-\theta_i e^{\theta_j}_{j,1})\boldsymbol{R_{j,2}}(F,F)\Bigg)\Bigg]\Bigg] + \\ (1-\mu_{-j,1})\Bigg[&\boldsymbol{R_{j,1}} - c(e^{\theta_j}_{j,1}) + \delta\Bigg[b\tilde{e}^b_{-j}\Bigg(\theta_j e^{\theta_j}_{j,1}\boldsymbol{R_{j,2}}(S,S) + (1-\theta_j e^{\theta_j}_{j,1})\boldsymbol{R_{j,2}}(F,S)\Bigg) + \\ &+ (1 - b\tilde{e}^b_{-j})\Bigg(\theta_j e^{\theta_j}_{j,1}\boldsymbol{R_{j,t}}(S,F) + (1-\theta_j e^{\theta_j}_{j,1})\boldsymbol{R_{j,2}}(F,F)\Bigg)\Bigg]\Bigg] \end{aligned} \tag{A.21}$$

Further, we define the *ex-post* payoff function π :

$$\begin{aligned} \pi_j(e^{\theta_j}_{j,1}, \theta_j, \tilde{e}^{\theta_{-j}}_{-j,1}, g) = &\boldsymbol{R_{j,1}} - c(e^{\theta_j}_{j,1}) + \delta\Bigg[g\tilde{e}^g_{-j}\Bigg(\theta_j e^{\theta_j}_{j,1}\boldsymbol{R_{j,t}}(S,S) + (1-\theta_j e^{\theta_j}_{j,1})\boldsymbol{R_{j,2}}(F,S)\Bigg) + \\ &+ (1 - g\tilde{e}^g_{-j})\Bigg(\theta_j e^{\theta_j}_{j,1}\boldsymbol{R_{j,2}}(S,F) + (1-\theta_j e^{\theta_j}_{j,1})\boldsymbol{R_{j,2}}(F,F)\Bigg)\Bigg] \end{aligned} \tag{A.22}$$

and hence the expression A.21 can be rewritten as:

$$\Pi_j(e^{\theta_j}_{j,1}, \theta_{-j}, \tilde{e}^{\theta_{-j}}_{-j,1}, \theta_{-j}) = \sum_{\theta_{-j} \in \{g,b\}} \pi_j(e^{\theta_j}_{j,1}, \theta_j, \tilde{e}^g_{-j,1}, \theta_{-j}) f(\theta_{-j}|\theta_j) \tag{A.23}$$

where $f(\theta_{-j}|\theta_j)$ is the conditional density of j's types.

Further, we define the incremental *ex-post* payoffs by:

$$\Delta\pi_j(e_j^{\theta_j}, e_j^{\prime\theta_j}, \theta_j; \tilde{e}_{-j}^{\theta_{-j}}, \theta_{-j}) \equiv \pi_j(e_j^{\theta_j}, \theta_j; \tilde{e}_{-j}^{\theta_{-j}}, \theta_{-j}) - \pi_j(e_j^{\prime\theta_j}, \theta_j; \tilde{e}_{-j}^{\theta_{-j}}, \theta_{-j}) \tag{A.24}$$

Given the assumptions on the cost function, the *ex-post* payoff function satisfies the following property as in Mason and Valentinyi, (2007)

P1 The payoff function $\pi_j : [0,1] \times \{g, b\} \to \mathbf{R}$ is bounded and measurable, and (upper semi-) continuous in own action. The types have conditional densities w.r.t the Lebesgue measure. The conditional density of θ_{-j} given θ_j , $f(\theta_{-j}|\theta_j)$ is strictly positive.

Given this property, we proceed to present conditions U1-U3 and D1-D2 as in Mason and Valentinyi, (2007). These conditions will enable us to prove uniqueness of the equilibrium.

U1 Uniformly Positive Sensitivity to Own Action and Type. There is a $\gamma \in (0, \infty)$ s.t. **for all** $e_{j,1}^{\theta_j} \geq e_{j,1}^{\prime\theta_j}$,$\theta_j \geq \theta_j'$, $\tilde{e}_{-j}^{\theta_{-j}}$,$\theta_{-j}$

$$\underbrace{\Delta\pi_j(e_{j,1}^{\theta_j}, e_{j,1}^{\prime\theta_j}, \theta_j; e_{-j,1}^{\theta_{-j}}, \theta_{-j}) - \Delta\pi_j(e_{j,1}^{\theta_j}, e_{j,1}^{\prime\theta_j}, \theta_j'; \tilde{e}_{-j}^{\theta_{-j}}, \theta_{-j})}_{\equiv \Delta_{\theta_j}\Delta u\left(e_{j,1}^{\theta_j}, e_{j,1}^{\prime\theta_j}, \tilde{e}_{-j,1}, \theta_j, \theta_{-j}\right)} \geq \gamma(e_{j,1}^{\theta_j} - e_{j,1}^{\prime\theta_j}) d_T(\theta_j, \theta_j'). \tag{A.25}$$

We assume $d_T(\theta_j, \theta_j') = |\theta_j - \theta_j'|$

Following the notation above and expression A.22, the ex-post incremental payoff for seller j (IP_j) for $e_j^{\theta_j} > e_j^{\prime\theta_j}$ is given by

$$\begin{aligned}
&\Delta\pi_j(e_{j,1}^{\theta_j}, e_{j,1}^{\prime\theta_j}, \theta_j; e_{-j,1}^{\theta_{-j}}, \theta_{-j}) = \\
&\quad = -\left(c\left(e_{j,1}^{\theta_j}\right) - c\left(e_{j,1}^{\prime\theta_j}\right)\right) + \delta\Bigg(E\left[\boldsymbol{R_{j,2}}\left(y_j, y_{-j}\right) \middle| e_{j,1}^{\theta_j}, \tilde{e}_{-j,1}^{\theta_{-j}}, \boldsymbol{\theta}\right] - E\left[\boldsymbol{R_{j,2}}\left(y_j, y_{-j}\right) | e_{j,1}^{\prime\theta_j}, \tilde{e}_{-j,1}^{\theta_{-j}}, \boldsymbol{\theta}\right. \\
&\quad = -\left(c\left(e_{j,1}^{\theta_j}\right) - c\left(e_{j,1}^{\prime\theta_j}\right)\right) + \delta\theta_j(e_{j,1}^{\theta_j} - e_{j,1}^{\prime\theta_j})\Bigg(\theta_{-j}\tilde{e}_{-j,1}^{\theta_{-j}}\left(\boldsymbol{R_{j,2}}\left(S,S\right) - \boldsymbol{R_{j,2}}\left(F,S\right)\right) + \\
&\qquad\qquad + \left(1 - \theta_{-j}\tilde{e}_{-j,1}^{\theta_j}\right)\left(\boldsymbol{R_{j,2}}\left(S,F\right) - \boldsymbol{R_{j,2}}\left(F,F\right)\right)\Bigg)
\end{aligned}$$

It follows then that

$$\begin{aligned}
\Delta_{\theta_j}\Delta u\left(e_{j,1}^{\theta_j}, e_{j,1}^{\prime\theta_j}, \tilde{e}_{-j,1}^{\theta_{-j}}, \theta_j, \theta_{-j}\right) =& \delta(\theta_j - \theta_j')(e_{j,1}^{\theta_j} - e_{j,1}^{\prime\theta_j})\Bigg(\theta_{-j}\tilde{e}_{-j,1}^{\theta_{-j}}\left(\boldsymbol{R_{j,2}}\left(S,S\right) - \boldsymbol{R_{j,2}}\left(F,S\right)\right) + \\
&+ \left(1 - \theta_{-j}\tilde{e}_{-j,1}^{\theta_j}\right)\left(\boldsymbol{R_{j,2}}\left(S,F\right) - \boldsymbol{R_{j,2}}\left(F,F\right)\right)\Bigg)
\end{aligned}$$

Proving that condition **U1** holds is equivalent to proving that $\exists\ \gamma \in (0,\infty)$ s.t. **for all** $e^{\theta_j}_{j,1} \geq e^{'\theta_j}_{j,1}$,$\theta_j \geq \theta'_j$, $\tilde{e}^{\theta_{-j}}_{-j}$, θ_{-j}

$$\underbrace{\delta\Bigg(\theta_{-j}\tilde{e}^{\theta_j}_{-j,1}\Big(\boldsymbol{R_{j,2}}\left(S,S\right) - \boldsymbol{R_{j,2}}\left(F,S\right)\Big) + \Big(1-\theta_{-j}\tilde{e}^{\theta_{-j}}_{-j,1}\Big)\Big(\boldsymbol{R_{j,2}}\left(S,F\right) - \boldsymbol{R_{j,2}}\left(F,F\right)\Big)\Bigg)}_{\equiv A} \geq \gamma \tag{A.26}$$

$$A = \delta\left(\theta_{-j}\tilde{e}^{\theta_{-j}}_{-j,1}\Big(r^S_{j,2}H_{2,j}\left(S,S\right) - r^F_{j,2}H_{2,j}\left(F,S\right)\Big) + \Big(1-\theta_{-j}\tilde{e}^{\theta_{-j}}_{-j,1}\Big)\left(r^S_{j,2}H_{2,j}\left(S,F\right) - r^F_{j,2}H_{2,j}\left(F,F\right)\right)\right) \tag{A.27}$$

The measure for the success probability $r = \mu g\tilde{e}^g + (1-\mu)b\tilde{e}^b$ is bounded above by $\bar{r} = g$ and bounded below by $\underline{\mathrm{r}} = b\underline{\mathrm{e}}$. Since the $H_{2,j}(r_{j,1}, r_{-j,1})$ is monotonically increasing in the first variable and decreasing in the second, we can define the lower bound for the $\underline{\mathrm{H}}_{2,j} = H_{2,j}(\underline{\mathrm{r}}, \bar{r})$. The upper bound would be then $\bar{H}_{2,j} = H_{2,j}(\bar{r}, \underline{\mathrm{r}})$. It is straightforward then that also A is bounded.

In what follows, we will need to determine a lower bound for A given the parameters N, g, b, δ. We call this lower bound γ. In the following Theorem 1 we will numerically determine values for γ together with other boundaries, for given parameter values N, g, b, δ.

U2 Lipschitz Continuity in Own Action There is an $\omega \in (0,\infty)$ s.t. **for all** $e^{\theta_j}_{j,1} \geq e^{'\theta_j}_{j,1}$,$\theta_j \geq \theta'_j$, $\tilde{e}^{\theta_{-j}}_{-j,1}$, $\theta_{-j,1}$

$$|\Delta\pi_j(e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1}, \tilde{e}^{\theta_{-j}}_{-j,1}, \theta_j, \theta_{-j})| \leq \omega(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1}).$$

The cost function and marginal cost function are monotonically increasing and bounded on the domain $[0,1]$, $\max\{c(1), c'(1)\} < C$. Hence given any two actions $e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1} \in [0,1]$, $e^{\theta_j}_{j,1} > e^{'\theta_j}_{j,1}$ there exists an $e^{\tilde{\theta}_j}_{j,1} \in \{e^{'\theta_j}_{j,1}, e^{\theta_j}_{j,1}\}$ such that $c'(e^{\tilde{\theta}_j}_{j,1})(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1}) = c\left(e^{\theta_j}_{j,1}\right) - c\left(e^{'\theta_j}_{j,1}\right)$. On the other hand, we know that $A \leq H_{2,j}(S,F) \leq H_{2,j}(\bar{r}, \underline{\mathrm{r}}) \equiv \bar{H}_{2,j}$. It hence follows that:

$$\begin{aligned}
|\Delta\pi_j(e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1}, \tilde{e}^{\theta_{-j}}_{-j}, \theta_j, \theta_{-j})| =& |-(c\left(e^{\theta_j}_{j,1}\right) - c\left(e^{'\theta_j}_{j,1}\right)) + \delta\theta_j(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1})A| \\
&\leq |-(c\left(e^{\theta_j}_{j,1}\right) - c\left(e^{'\theta_j}_{j,1}\right))| + |\delta\theta_j(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1})A| \\
&\leq c'(e^{\tilde{\theta}_j}_{j,1})(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1}) + \delta\theta_j\bar{H}_{2,j}(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1}) \\
&= \left(c'(e^{\tilde{\theta}_j}_{j,1}) + \delta\theta_j\bar{H}_{2,j}\right)(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1}) \\
&\leq \left(C + \delta\theta_j\bar{H}_{2,j}\right)(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1})
\end{aligned}$$

We can now define the parameter $\omega = C + \delta\theta_j \bar{H_{2,j}}$ s.t.

$$|\Delta\pi_j(e^{\theta_j}_{j,1}, e'\theta_{j\,j,1}, \tilde{e}^{\theta_{-j}}_{-j}, \theta_j, \theta_{-j})| \leq \omega(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1})$$

U3 Uniformly Bounded Sensitivity to Opponent's Action. There is a $\kappa \in (0, \infty)$ s.t. **for all** $e^{\theta_j}_{j,1} \geq e^{'\theta_j}_{j,1}$,$\theta_j \geq \theta'_j$, $\tilde{e}^{\theta_{-j}}_{-j,1}$, θ_{-j}

$$|\underbrace{\Delta\pi_j(e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1}, \theta_j; \tilde{e}^{\theta_{-j}}_{-j,1}, \theta_{-j}) - \Delta\pi_j(e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1}, \theta_j; \tilde{e}^{'\theta_{-j}}_{-j,1}, \theta_{-j})}_{|\Delta_{\tilde{e}^{\theta_{-j}}_{-j,1}} \Delta\pi_j(e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1}, \theta_j; \tilde{e}^{\theta_{-j}}_{-j,1}, \theta_{-j})|}| \leq \kappa(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1})$$

$$|\Delta_{\tilde{e}^{\theta_{-j}}_{-j,1}} \Delta\pi_j(e^{\theta_j}_{j,1}, e^{'\theta_j}_{j,1}, \theta_j; \tilde{e}^{\theta_{-j}}_{-j,1}, \theta_{-j})| = \Bigg|\delta\Bigg(\Big(\boldsymbol{R_{j,2}}(S,S) - \boldsymbol{R_{j,2}}(F,S)\Big) - \\ -\Big(\boldsymbol{R_{j,2}}(S,F) - \boldsymbol{R_{j,2}}(F,F)\Big)\Bigg)\theta_j\theta_{-j}\left(\tilde{e}^{\theta_{-j}}_{-j,1} - e^{'\theta_{-j}}_{-j,1}\right)\left(e^{\theta_j}_{j,1} - e^{'\theta_j}_{j,1}\right)$$

Similar reasoning as in **U1** applies to conclude that the term

$$\underbrace{\left|\delta\Bigg(\Big(\boldsymbol{R_{j,2}}(S,S) - \boldsymbol{R_{j,2}}(F,S)\Big) - \Big(\boldsymbol{R_{i,2}}(S,F) - \boldsymbol{R_{i,2}}(F,F)\Big)\Bigg)\theta_i\theta_{-i}\left(\tilde{e}^{\theta_{-j}}_{-j} - \tilde{e}^{'\theta_{-j}}_{-j}\right)\right|}_{\equiv K} \tag{A.28}$$

is bounded. We denote the upper bound of this term by κ. In what follows, we will need to determine an upper bound for K given the parameters N, g, b, δ. In Theorem 4 we will numerically determine values for κ together with other boundaries, for given parameter values N, g, b, δ.

D1 There is a $\iota \in (0, \infty)$ s.t. for any $\theta_k > \theta'_k$ and $k \in \{j, -j\}$ $\sqrt{\mathcal{I}(\theta_k, \theta'_k)} \leq \iota d_T(\theta_k, \theta'_k)$, where

$$\mathcal{I}(\theta_k, \theta'_k) \equiv Var_{T_{-k}}\left(\frac{f(\theta_{-k}|\theta_k) - f(\theta_{-k}|\theta'_k)}{f(\theta_{-k}|\theta_k)}\right)$$

This is a condition on the *Fisher's information* of a player's type about the types of the opponents. Note that in our case, we assume *independence* of types. In other words, $f\left(\theta_{-k}\middle|\theta_k = g\right) = f\left(\theta_{-k}\middle|\theta_k = b\right)$ which implies that $\mathcal{I}(\theta_k, \theta'_k) = 0$. Therefore, the condition

$$\sqrt{\mathcal{I}(\theta_k, \theta'_k)} \leq \iota\,(g - b)$$

holds for any $\iota \in (0, \infty)$.

D2 There is a $\nu \in [0, \infty)$ s.t. $f_{-j}(\theta_{-j}|\theta_j) \leq \nu$ for all $j \neq -j$. This is a condition on the (ex-ante) heterogeneity of types. We are looking for some $\nu \in [0, \infty)$ such that $f_{-j}\left(\theta_{-j}\middle|\theta_j\right) \leq \nu \ \forall j, -j$. In terms of initial beliefs, this implies for $k \in \{j, -\}$

$$\begin{cases} \mu_k & \leq \nu \\ 1 - \mu_k & \leq \nu \end{cases}$$

$\Rightarrow$ Any $\nu \in [1, \infty]$ satisfies this condition.

Theorem 4: *If assumptions U1-U3 and D1-D2 hold, and if*

$$\gamma > \iota\omega + \upsilon\kappa \tag{A.29}$$

then the best response (BR) correspondence is a contraction, and hence there is a unique equilibrium of the Bayesian game. Furthermore, this equilibrium is in monotone pure strategies.

In compliance with conditions **U1-U3** and **D1-D2**, we now determine value of the parameters $N, g, b,$ and δ (to determine the values for γ and κ) for which the relation A.29 holds. We recall that:

$$\omega = C + \delta\theta_j \bar{H_{2,j}}, \text{where } \ C \leq c'(1)$$

$$\iota = \epsilon$$

$$\nu = 1$$

where we have chosen ϵ to be arbitrarily small, $\epsilon \to 0$.
We investigate now how the maximal value of K (henceforth denoted by $\bar{K} = \max K$)in expression A.28 and the minimal value of A in A.26 (henceforth denoted by $\underline{\text{A}} = \min A$) vary with the parameters N, g, b and δ. Our target is to pin down restrictions for these parameters for which $\bar{K} < \underline{\text{A}}$

A.4.9 Proof of Proposition 5

Numerical.

A.5 The Value of Reputation in Infinite Horizon

Estimating the effect of reputation on revenues and sales is a key question for both market participants (sellers) and market makers or regulators (eBay, etc). The latter need to design a way for buyers to have information about the reliability of sellers, while reliable sellers will be reliable depending on how outcomes and information might affect their revenues. Reputation mechanisms are considered to be a solution to the market failures arising from the different sources of inherent asymmetric information. Concretely, and in the spirit of Akerlof, (1970), a well-functioning reputation / feedback mechanism might help welfare improving trades that where previously not happening to materialize, and in the process allow "good"/ "truthful" sellers to increase their revenues.

In this section we are interested in quantifying the bias that might arise in the *reduced form* estimation of the impact of reputation on short-run seller revenues when reputation building and strategic behavior are not incorporated into the identification strategy. Although recent empirical studies using both longitudinal (panel) data on sellers and "field" experiments, such as Resnick et al., (2006) and Cabral and Hortaçsu, (2010), have gone a long way in trying to identify such effect, we argue that their omission of the above two behaviors might render their estimates misleading.

First, strategic behaviors can generate reputations of competing sellers that are *correlated* across time and a determinant of revenues. Under such scenario, not accounting for competitors' information will bias the estimation of the reputation-elasticity of revenues. The idea is simple: in our setting, when buyers select an auction and bid, it is in their best interest to judge the trustworthiness of *all* sellers through the available public information. The publicly observed reputation of competitors then helps seller j predict the behavior of *future potential* buyers and other seller, behaviors which will determine her revenues. Seller j's effort strategies, therefore, will take into account how current beliefs will affect the updating process of buyers[2], and how this will affect the pool of future buyers that bid in her auction.[3]

Second, Resnick et al., (2006) does not control for the kind of reputation building behavior that has been characterized in seminar works such as Holmstrom, (1999).

We will frame the analysis within an infinite horizon version of the game described in the previous sections; this will allow us to incorporate the main mechanisms into an environment where "professional" sellers care about the medium-long term when making decisions. A potential drawback with the benchmark environment is is that it is stripped from many relevant characteristics of on-line markets and industries in general, such as a large number of sellers or the possibility to exit and re-enter the market. Nonetheless, it still allows us to do a first quantification of the endogeneity problem.

In a second step, we will extend the benchmark model by incorporating more seller into the market, and maintaining the assumption that both sellers and buyers behave strategically. In order to keep the problem computationally tractable, Our approach will follow the recent developments in computable models of industry dynamics, recently reviewed by Doraszelski and Pakes, (2007). In these lines, in the multiple seller

[2] Since seller $-j$ acts in the same way, seller's j best response to her competitors actions and buyers behavior is a best response to current reputation levels. This is simply the main insight from a Markov equilibrium.

[3] Perhaps the strongest assumption here is both the rational expectation of sellers and buyers and their optimal use of all the available information. One could argue that such processing capacity is not realistic, or that the formation of expectations by sellers follows a more reduced form version of the fully rational one. Such considerations, though certainly valid and perhaps even accurate, are outside the scope of this paper.

setup we assume that each seller only

A.5.1 Strategic Competition: Recursive Formulation

We first present the infinite horizon version of the problem presented in section 1.3, casted in terms of reputation (beliefs about seller types) as well as the type itself as state variables.

Belief $\boldsymbol{\mu}_\tau \equiv \{\mu_{j,\tau}, \mu_{-j,\tau}\}$ are a *sufficient statistic* about the sequence of outcomes up to $\tau - 1$. Given that the outcomes of sellers are (conditionally) independent, the recursive formulation of problem for seller j is

$$\begin{aligned} V_j^{comp}(\mu_j, \mu_{-j}; \theta) = \quad & \max_{e_j^\theta} \Bigg\{ \boldsymbol{R}(\mu_j, \mu_{-j}, \boldsymbol{e}^*) - c\left(e_j^\theta\right) + \\ & +\delta \sum_{y_j} \sum_{y_{-j}} p\left(y_j, y_{-j} | e_j^\theta, r_{-j}\right) V_j^{comp}\left(\mu_j', \mu_{-j}'; \theta\right) \Bigg\} \\ s.t. \qquad & e_j^\theta \in [0,1] \end{aligned} \tag{A.30}$$

where $\boldsymbol{e}^* \equiv \left\{e_j^{g*}, e_j^{b*}, e_{-j}^{g*}, e_{-j}^{b*}\right\}$ and $V_j^{comp}(\cdot)$ is the continuation value for seller j after the outcomes of both sellers, $\{y_j, y_{-j}\}$, which are publicly observable. Recall that, due to Bayesian learning, μ_j' and μ_{-j}' are functions of $\boldsymbol{e}^*$, $\boldsymbol{\mu}$ and y_j, y_{-j}. In addition, r_{-j} is a function of both μ_{-j} and $\boldsymbol{e}^*_{-j}$. The notation in (A.30) implies that $\boldsymbol{e}^*$ are effort decisions as perceived by other sellers and buyers, which will be confirmed in an equilibrium: since all players have rational expectations, each one can solve the problem of the other and therefore "guess" an effort choice.

No Reputation and Perfect Reputation as Absorbing States

It is important to note that, due to the assumption about sellers / buyers being Bayesian learners, the Markov process has two *absorbing states*: $\mu = 0$ and $\mu = 1$. That is,

$$\mu_{j,t} \equiv P\left(\theta_j = g | \mu_{j,t-1} = 0, y_{j,t-1}\right) = 0 \quad \forall\, t, y_{j,t-1}, \mu_{-j,t} \tag{A.31}$$

and

$$\mu_{j,t} \equiv P\left(\theta_j = g | \mu_{j,t-1} = 1, y_{j,t-1}\right) = 1 \quad \forall\, t, y_{j,t-1}, \mu_{-j,t} \tag{A.32}$$

Note that expressions (A.31) and (A.32) hold for *any* effort level of both sellers. Given the properties of the cost function, the optimal effort under such states is the minimum effort $\underline{e}$. It is then straightforward, given that $c(\underline{e}) = 0$, to compute the value for a seller at these states:

Proposition 6 *The continuation values at each absorbing state satisfy*

$$V_j^{comp}(1,0;\cdot) > V_j^{comp}(1,1;\cdot) > V_j^{comp}(0,0;\cdot) > V_j^{comp}(0,1;\cdot) \tag{A.33}$$

Proof. Each term in the relation (A.33) represents the value of selling under competition when the type-beliefs of *both* sellers have converged to either 0 or 1. Such beliefs need not coincide with the *true* type of sellers; indeed, the value at an absorbing state / belief for seller j is *independent* of her true type. Then, we have

$$V_j^{comp}(0,0;\theta_j) = \frac{\mathbb{E}\left(b_j^{(2)}\middle|\mu_j=0,\mu_{-j}=0\right)}{1-\delta} = \frac{r_j(0,0)\mathbb{E}\left(v_j^{(2)}\middle|\mu_j=0,\mu_{-j}=0\right)}{1-\delta} \tag{A.34}$$

$$V_j^{comp}(0,1;\theta_j) = \frac{\mathbb{E}\left(b_j^{(2)}\middle|\mu_j=0,\mu_{-j}=1\right)}{1-\delta} = \frac{r_j(0,1)\mathbb{E}\left(v_j^{(2)}\middle|\mu_j=0,\mu_{-j}=1\right)}{1-\delta} \tag{A.35}$$

$$V_j^{comp}(1,0;\theta_j) = \frac{\mathbb{E}\left(b_j^{(2)}\middle|\mu_j=1,\mu_{-j}=0\right)}{1-\delta} = \frac{r_j(1,0)\mathbb{E}\left(v_j^{(2)}\middle|\mu_j=1,\mu_{-j}=0\right)}{1-\delta} \tag{A.36}$$

$$V_j^{comp}(1,1;\theta_j) = \frac{\mathbb{E}\left(b_j^{(2)}\middle|\mu_j=1,\mu_{-j}=1\right)}{1-\delta} = \frac{r_j(1,1)\mathbb{E}\left(v_j^{(2)}\middle|\mu_j=1,\mu_{-j}=1\right)}{1-\delta} \tag{A.37}$$

where $r_j(0,0) = r_j(0,1) = \underline{e} * b < r_j(1,0) = r_j(1,1) = \underline{e} * g$ are the perceived probabilities of successfull delivery. Recall that the properties of the function $H_{2j}(\mu_j,\mu_{-j}) \equiv \mathbb{E}\left(v_j^{(2)}\middle|\mu_j,\mu_{-j}\right)$ (the expected 2^{nd} highest valuation in seller j's auction when type beliefs are (μ_j,μ_{-j}) are such that $H_{2j}(\mu_j,\mu_{-j})$ is increasing in μ_j, decreasing in μ_{-j} and $H_{2j}(\cdot,\cdot) = H_{2-j}(\cdot,\cdot)$. The result then follows inmediately. ■

Relation (A.33) says that, as one would expect, the highest value for seller j of selling under competition is realized when the belief other market participants have about her are maximal (i.e. 1) and the beliefs about her competitor are minimal. In such case, she can enjoy the returns to a perfect reputation even by putting minimum effort, without fearing about her competitor. On the other side of the spectrum, her continuation value is minimal when her belief is minimal and her competitor enjoys perfect reputation. For the middle cases, the relation $V_j^{comp}(1,1;\cdot) > V_j^{comp}(0,0;\cdot)$ arises since, although the expected second highest valuation is the same when $(\mu_j,\mu_{-j}) = (0,0)$ and $(\mu_j,\mu_{-j}) = (1,1)$, a higher perceived probability of successful delivery $r_j(\cdot,\cdot)$ incentivizes each buyer to bid higher.

Note that whether relation (A.33) is enough in order for a bad seller to try to disguise himself as a good seller in any state, will depend on the marginal value of effort at different type beliefs.

Optimal Effort and the Euler Equation

As in the two period case, the effort best response of sellers $j = 1, 2$ and types $\theta \in g, b$ are implicitly characterized by the system of FOCs:

$$\begin{aligned} c'\left(e_j^\theta\right) &= \delta\theta\Bigg(r_{-j}\left(V_j^{comp}\left(\mu_j'(S), \mu_{-j}'(S); \theta\right) - V_j^{comp}\left(\mu_j'(F), \mu_{-j}'(S); \theta\right)\right) + \\ &\quad + (1 - r_{-j})\left(V_j^{comp}\left(\mu_j'(S), \mu_{-j}'(F); \theta\right) - V_j^{comp}\left(\mu_j'(F), \mu_{-j}'(F); \theta\right)\right)\Bigg) \\ &\quad j = 1, 2\,;\ \theta = g, b \end{aligned} \tag{A.38}$$

where $\mu_j'(y)$ is the updated reputation after an outcome y in the current transaction.

From the discussion in the previous subsection, it is clear that whenever $\mu_j = 0$ or $\mu_j = 1$, the right hand side of (A.39) is 0, for any μ_{-j}. Therefore, in those cases $e_j^\theta = \underline{e}$. In the rest of the state space, the intuition for (A.39) is relatively straightforward. Given that effort decisions affect directly the probability of a successful outcome, the marginal benefit of effort is given by the difference in continuation value between a success and a failure, for all the possible outcomes of the competitor, weighted by the marginal change in the probability of a success. A seller will be willing to increase her effort as long as the (discounted) incremental difference in the continuation value, weighted by the marginal effect of effort on the outcome probabilities θ, is big enough. Seller j chooses effort strategically taking into account two pieces of information: (i) what buyers perceive about other types of sellers, which will be part of the updating process leading to μ_j', and (ii) how the "intensity" of competition (given by the beliefs about her competitor's type) will affect her expected revenues, which affects both r_{-j} and μ_{-j}' in the RHS of (A.39). In this setting, the expression for the Euler equation for a seller j of type θ is non-standard. The reason is the following: an effort decision after auction t , e_t, will have a probabilistic effect on the state tomorrow, μ'. Importantly, however, this change in the distribution of μ' will affect future states through two channels: future decisions of effort based on μ' which will shape the distribution of μ'', μ''', etc, and through the updating process, which will mean that the effects of e_t will persist.

In order to save on notation below, lets consider, without loss of generality, the case in which the outcome y can take a continuum of values. When then can write the FOC (A.39) for seller j and type θ_j as

$$-c'\left(e_j^\theta\right) + \delta \int_{y_{-j}} \int_{y_j} V^{comp}\left(\mu_j', \mu_{-j}'; \theta\right) \frac{\partial f\left(y_j | e_j, \theta\right)}{\partial e_j}\Big|_{e_j = e_j^\theta} f\left(y_{-j} | \boldsymbol{e}_{-\boldsymbol{j}}^*\right) \mathrm{d}y_j \mathrm{d}y_{-j} = 0 \tag{A.39}$$

where e^θ is the optimal effort strategy for a seller of type θ, and $f(y|e, \theta)$ is the probability density for the outcome y conditional on e and θ.

If we repeatedly replace $V^{comp}(\cdot)$ in the expression above, we get

$$
\begin{aligned}
c'\left(e^{\theta}\right) &= \delta \int_{\boldsymbol{y}} \frac{\partial f\left(y_j|e_j,\theta\right)}{\partial e_j}\Big|_{e_j=e_j^{\theta}} \pi_j\left(\boldsymbol{\mu'}, \boldsymbol{e'}^{*}; e'\right) \mathrm{d}\boldsymbol{y} + \\
&+\delta^2 \int_{\boldsymbol{y}} \frac{\partial f\left(y_j|e_j,\theta\right)}{\partial e_j}\Big|_{e_j=e_j^{\theta}} \int_{\boldsymbol{y'}} \pi_j\left(\boldsymbol{\mu''}, \boldsymbol{e''}^{*}; e''\right) f\left(\boldsymbol{y'}|\boldsymbol{e''},\theta\right) \mathrm{d}\boldsymbol{y'}\mathrm{d}\boldsymbol{y} + \\
&+ \dots
\end{aligned} \tag{A.40}
$$

where $\boldsymbol{y} \equiv \{y_j, y_{-j}\}$, $\boldsymbol{\mu} \equiv \{\mu_j, \mu_{-j}\}$, $\boldsymbol{e} \equiv \{\boldsymbol{e_j}, \boldsymbol{e_{-j}}\}$ and $\pi\left(\boldsymbol{\mu}, \boldsymbol{e}^{*}; e\right) = \boldsymbol{R}\left(\mu, \boldsymbol{e}^{*}\right) - c\left(e\right)$ is the stage-payoff. In a Markov Perfect equilibrium, of course, the effort decisions $\boldsymbol{e}$ will be a function of current beliefs $\boldsymbol{\mu}$.

Although in this case the derivation of the Euler eq. does not require to work explicitly with the derivative of V^{comp} with respect to the state μ, each seller *does* care about the slope of the value function when deciding an effort level[4]. Therefore, it is still useful to have a look at the slope of $V^{comp}\left(\mu_j.\mu_{-j}\right)$. Assuming differentiability of $V^{comp}\left(\boldsymbol{\mu}; \cdot\right)$, the *envelope condition*, describing how the *maximum* value for a seller changes when the state μ varies, is given by

$$
\begin{aligned}
\frac{\partial V^{c}\left(\boldsymbol{\mu};\theta\right)}{\partial \mu_j} &= \frac{\partial \boldsymbol{R}\Big(\boldsymbol{\mu}, \boldsymbol{e}\left(\boldsymbol{\mu}\right)\Big)}{\partial \mu_j} + \\
&+\frac{\partial e_j^{\theta}\left(\boldsymbol{\mu}\right)}{\partial \mu_j}\left(- c'\left(e_j^{\theta}\left(\boldsymbol{\mu}\right)\right) + \delta \int_{y_{-j}} \int_{y_j} V^{c}\left(\boldsymbol{\mu'};\theta\right) \frac{\partial f\left(y_j|e_j,\theta\right)}{\partial e_j}\Big|_{e_j=e_j^{\theta}} f\left(y_{-j}|\boldsymbol{e^{*}_{-j}}\right) \mathrm{d}y_j \mathrm{d}y_{-j}\right) \\
&+\delta \int_{y_{-j}} \int_{y_j} \frac{\partial V^{comp}\left(\boldsymbol{\mu'};\theta\right)}{\partial \mu'_j} \frac{\partial \mu'_j}{\partial \mu_j} f\left(y_j|e_j^{\theta},\theta_j\right) f\left(y_{-j}|\boldsymbol{e^{*}_{-j}}\right) \mathrm{d}y_j \mathrm{d}y_{-j}
\end{aligned}
$$

Note that the term in parenthesis in the second row is equal to 0 at an optimum. Therefore, we have

$$
\frac{\partial V^{comp}\left(\boldsymbol{\mu};\theta\right)}{\partial \mu_j} = \frac{\partial \boldsymbol{R}\Big(\boldsymbol{\mu}, \boldsymbol{e}\left(\boldsymbol{\mu}\right)\Big)}{\partial \mu_j} + \delta \mathbb{E}\left(\frac{\partial V^{comp}\left(\boldsymbol{\mu'};\theta\right)}{\partial \mu'_j} \frac{\partial \mu'_j}{\partial \mu_j} \Bigg| e_j^{\theta}, \theta_j, \boldsymbol{e^{*}_{-j}}\right) \tag{A.41}
$$

The second term on the right-hand side of expression (A.41) is "non-standard", and represents the second channel through which current effort decisions generate future benefits. One interpretation of this can be the following: in addition to the direct impact on *current* revenues, a marginal change in the type belief today μ affects the future learning dynamics. This impact is very particular of a Bayesian learning environment. Therefore, such change will affect *all* the future profits for the seller, since all the future

[4]Technically, the reason is that, although the seller can only affect the density (i.e. weights) with which the future expected value of reputation is computed, the optimal decision implies shifting probability mass from certain outcomes (y_j) to other outcomes. In other words, the derivative of the density function $f\left(y|e,\theta\right)$ with respect to e will be be negative for certain values of y_j and positive for others. This implies that seller j will decide the effort level by "shifting" probability mass such that, marginally, the weighted change in value is the same as the change in the cost of effort.

generations of buyers will shape their beliefs (and therefore their selection and bidding strategies) accordingly[5]. The link with the Euler equation (A.40) is then clear: an optimal effort strategy (one that maximizes the value for a seller) affects the distribution of beliefs tomorrow, which triggers a change in *expected* short-run nrevenues and a new"learning" path which will affect all future expected profits. The latter impact appears *independently* of the effort decision tomorrow.

A.5.2 Existence of a Markov Perfect Equilibrium

We focus on Markov Perfect (Bayesian) Equilibibrium concept, as defined in section 1.3.5. Existence in the dynamic setting is, however, a delicate issue. Given that sellers' actions are continuous, we follow Adlakha, Johari, and Weintraub, (2010) and Escobar, (2011) to argue the existence of an equilibrium. Their main results imply the existence of a MPE provided the sets of actions are compact, the set of states is countable, the period payoffs are upper semi-continuous in the action profiles and lower semi-continuous in the competitors' actions and the transition function depends continuously on sellers' actions.

A.5.3 The (Short-Run) Reputation-Elasticity of Revenues

In our dynamic setting, when deciding about effort levels, sellers care about the marginal effect of reputation on their *lifetime* income from future transactions. As shown in equation (A.41), the bayesian learning structure implies that, at an equilibrium,

$$\frac{\partial V^{comp}\left(\boldsymbol{\mu};\theta\right)}{\partial \mu_j} \neq \frac{\partial \boldsymbol{R}\Big(\boldsymbol{\mu},\boldsymbol{e}\left(\boldsymbol{\mu}\right)\Big)}{\partial \mu_j} \tag{A.42}$$

The size of the difference between the left-hand side (LHS) and the right-hand side (RHS) in (A.42) will be determined by the slope of of the learning function w.r.t μ, that is, $\frac{\partial \mu'_j}{\partial \mu_j}$. This determines how much does a change in current reputation affects the value for a seller due to the impact of the learning structure on future revenues. The second term in the envelope condition (A.41) is an infinite sum of future revenues, weighted by such learning slope and the discount factor. On one hand, when the potential types of sellers are far appart (i.e. $g - b \approx 1$) the learning slope will tend to be relatively big compared to the case when seller types are not very different. On the other hand, however, when sellers are either very trustworthy or not trustworthy, types will be revealed quite fast, meaning that soon into the life of a seller, the reputaion μ will be either 0 or 1 forever.

The equilibrium of the infinite horizon game implies a particular form for the effort strategies; these are a function of current reputation and "deep" parameters, which include the discount factor δ, type values g, b , the number of buyers N in each period, and cost function parameters C. Therefore, we have that equilibrium (expected)

[5]Recall that, although buyers are short lived, they observe a summary of past outcomes, represented by μ.

revenues at the beginning of a period for seller j can be expressed as

$$\boldsymbol{R_j}\Big(\mu_j, \mu_{-j}, e_j(\boldsymbol{\mu};\Omega), e_{-j}(\boldsymbol{\mu};\Omega)\Big) = r_j\Big(\mu_j, e_j(\boldsymbol{\mu};\Omega)\Big) H_{2j}\Big(\mu_j, \mu_{-j}, e_j(\boldsymbol{\mu};\Omega), e_{-j}(\boldsymbol{\mu};\Omega)\Big) \tag{A.43}$$

where $\Omega \equiv \{\delta, g, b, N, C\}$, $\boldsymbol{\mu} \equiv \{\mu_j, \mu_{-j}\}$, and $e_j(\boldsymbol{\mu}) \equiv \left\{e_j^g(\boldsymbol{\mu}), e_j^b(\boldsymbol{\mu})\right\}$ are equilibrium effort strategies.

A natural meassure of the impact of reputation is the elasticity of expected revenues in period t for seller j with respect to her reputation in the same period. In equilibrium, such elasticity, which we denote $\eta_{\mu_j}^{R_j}$, depends on effort strategies and is given by (for simplicity, the notation abstract from the dependence on deep parameters):

$$\begin{aligned}\eta_{\mu_j}^{R_j} &\equiv \frac{\partial \boldsymbol{R_j}\Big(\mu_j, \mu_{-j}, e_j(\boldsymbol{\mu};\Omega), e_{-j}(\boldsymbol{\mu};\Omega)\Big)}{\partial \mu_j} \times \frac{\mu_j}{\boldsymbol{R_j}\Big(\mu_j, \mu_{-j}, e_j(\boldsymbol{\mu};\Omega), e_{-j}(\boldsymbol{\mu};\Omega)\Big)} \\ &= h\Big(\mu_j, \mu_{-j}; \Omega\Big)\end{aligned} \tag{A.44}$$

The crucial aspect is that $H_{2j}(\cdot)$ is a function of both sellers effort in equilibrium. This implies that the reputation-elasticity of revenues is potentially a function of *both* reputations, and therefore not constant if reputations evolve over time. In addition to the dependence on the competitor's reputation, and given the characterization results in proposition 5, the dependence onn the number of buyers N, is of interest too. As it has been shown, when the number of buyers in the market increases, their equilibrium behavior implies that sellers startegic incentives disappear. Therefore, we expect the nature of the bias that might arise from reduced form estimations of (A.44) to change with N too.

Reduced Form Estimation of $\eta_{\mu_j}^{R_j}$?

Recall that, conditional on sellers reputations, there are two sources of uncertainty in the environment we proposed: (i) the valuations that the N buyers have for the good, which are independently drawn each period from $U \sim [0,1]$, and (ii) the selection of auctions characterized in equilibrium by *mixed* strategies. We can write *realized* revenues in equilibrium for seller j at time t as

$$\boldsymbol{RR_{j,t}} = r_j(\mu_{j,t}, \mu_{-j,t})\, v_j^{(2)} \tag{A.45}$$

where $v_j^{(2)}$ is the *realized* second highest valuation in seller j auction. Importantly, $\boldsymbol{RR_{j,t}}$ and $(\mu_{j,t}, \mu_{-j,t})$ are the variables that a researcher can potentially observe.

Given that we assume sellers form rational expectations, and that conditional on the parameters of the game Ω srategies are a function only of $\boldsymbol{\mu}$, we can think of the expected revenues $\boldsymbol{R_{j,t}}$ as a linear projection of $\boldsymbol{RR_{j,t}}$ on the space of a particular type of functions of $\boldsymbol{\mu_t}$. In other words, we can decompose $\boldsymbol{RR_{j,t}}$ as

$$\boldsymbol{RR_{j,t}} = \boldsymbol{R_{j,t}} + \epsilon_{j,t} \tag{A.46}$$

where $\boldsymbol{R_{j,t}} = \mathbb{E}\left(\boldsymbol{RR_{j,t}}\middle|\boldsymbol{\mu_t}\right)$ and $\epsilon_{j,t}$ is a forecast error with

$$\mathbb{E}\left(\epsilon_j\middle|\boldsymbol{\mu_t}, \boldsymbol{e}\left(\boldsymbol{\mu_t}\right), \boldsymbol{\mu_{t-1}}, \boldsymbol{\ldots}\right) = \mathbb{E}\left(\epsilon_j\middle|\boldsymbol{\mu_t}, \boldsymbol{e}\left(\boldsymbol{\mu_t}\right)\right) = 0 \tag{A.47}$$

Note that, although the decomposition in (A.46) arises from a linear projection, $\boldsymbol{R_{j,t}}$ is potentially a non-linear function of $(\mu_{j,t}, \mu_{-j,t})$.

All recent empirical studies about the impact of reputation on revenues, such as Cabral and Hortaçsu, (2010) who uses both cross section as well as panel data for sellers, or Resnick et al., (2006) who use field experiment data in order to generate exogenous variation in reputation, estimate a specification similar to the following (log) linear expression for $\boldsymbol{RR_{j,t}}$ in order to try to uncover some information about η^R_μ:

$$\log \boldsymbol{RR}_{j,t} = \beta_0 + \beta_1 \log \mu_{j,t} + X_{j,t}\Gamma + \nu_{j,t} \tag{A.48}$$

where $\log \boldsymbol{RR}_{j,t}$ are the log-revenues[6] for seller j in period t; $\log \mu_{j,t}$ is the log-public reputation of seller j in period t; X_j is a set of other control variables potentially including seller's fixed characteristics, and $\nu_{j,t}$ is an error term.

Would a simple OLS estimate of β_1, $\hat{\beta}_1^{OLS}$ using either panel or experimental data on revenues and reputation, be enough to consistently estimate η^R_μ? Under general conditions, the answer is likely to be "no". In order to uncover η^R_μ through eq. (A.48), one needs to make at least two assumptions: (i) the reputation of seller j's competitors $\log \mu_{-j,t}$ does not directly influence seller j's revenues, and (ii) the way in which $\log \mu_{j,t}$ affects $\boldsymbol{RR}_{j,t}$ needs to be constant across μ_j. Under the structure of the simple market presented here, equilibrium revenues depend on effort strategies, which are not observable by the econometrician, and they are a function of *both* sellers reputations. In other words,

$$\nu_{j,t} = g\Big(, e_j\left(\boldsymbol{\mu}\right), e_{-j}\left(\boldsymbol{\mu}\right); \epsilon_{j,t}\Big) \tag{A.49}$$

In equilibrium, it will probably be the case that $Cov\Big(\log \mu_{j,t}, \nu_{j,t}\Big) \neq 0$: not only might $\mu_{j,t}$ and $\mu_{-j,t}$ be correlated through past and current effort, but the dependence of RR on $\mu_{j,t}$ is potentially not constant. The latter implies that even an exogenous source of variation in $\mu_{j,t}$, say from a field experiment as in Resnick et al., (2006), would not be enough to uncover most of the information about the elasticity.[7]

[6] Due to lack of time series data on past prices, Cabral and Hortaçsu, (2010) proxy revenues by the number of sales. For our purpose, however, they both measure similar aspects of trades.

[7] Resnick et al., (2006) generate such exogenous variation through an experiment in which a seller with high reputation creates an additional account under a different name. Under this new account, the auction characteristics are the same as the ones in the auctions under her original name, but reputation would be clearly different. The relevant parameter, which in essence is their counterpart of β can be computed as

$$\tilde{\beta} = \bar{\boldsymbol{RR}}_j^{original} - \bar{\boldsymbol{RR}}_j^{new}$$

The next relevant question is then: can a reduced form estimation such as (A.48) provide some useful information about η^R_μ, *even* when η^R_μ cannot be consistently identified by $\hat{\beta}_1^{OLS}$? Consider the unweighted *average* of $\eta^{R_j}_{\mu_j}$ across the whole state space (μ_j, μ_{-j})

$$\bar{\eta}_j = \int_{M_j} \int_{M_{-j}} h\Big(\mu_j, \mu_{-j}; \Omega\Big)\, d\,\mu_{-j}\, d\,\mu_j \tag{A.50}$$

A.5.4 Quantitative Experiment

In this section we quantify importance of the endogeneity problem described above, which can arise when strategic interactions and reputation building behavior are not taken into account in the reduced form estimation.

We first propose an algorithm to solve the infinite-horizon game, and present the quantitative properties of the approximated equilibrium (including the true elasticity of revenues) for different parameter values. We then calibrate a benchmark version using moments presented in Cabral and Hortaçsu, 2010, and use simulated data to run different regressions that allow us to analyze quantitatively the importance of omitting reputation building and strategic behavior from the reduced form model.

Computational Algorithm

In order to solve the dynamic game for a given set of parameter values, we propose an algorithm in the spirit of Doraszelski and Pakes, (2007) solving for the value functions of each seller and type, while searching for an equilibrium using the contraction properties of best responses. Some important aspects that the algorithm needs to take into account:

There are four key features of the dynamic game that must be taken into account when solving the problem numerically:

1. Since effort decisions are not observable, they do not affect the type-beliefs directly, but indirectly through the information (signal) revealed by the outcomes y_j. As described in the previous section, an effort decision $e_{j,t}$ at time t (i.e. conditional on the information set at time t) will have an impact on all the probabilities of future outcome sequences. This feature prevents us from using a numerical algorithm that works only through the Euler equations since these would be composed of an infinite sum of terms (i.e. future profits) weighted by the change in probabilities after an effort decision at time t.

2. Probabilities of outcomes (deliveries) and beliefs are both *subjective perceptions of seller j*, function of the best responses (BR) for all players, and these perceptions

.

must be confirmed in equilibrium. From seller j's point of view:

$$P\left(y_j', y_{-j}'|\theta_j, BR_j\left(\boldsymbol{e^*_{-j}}, \boldsymbol{s}\right)\right) = P\left(y_j'|\theta_j, e_j^\theta\right) \cdot P\left(y_{-j}'|\boldsymbol{e^*_{-j}}, \mu_{-j}\right) \tag{A.51}$$

where the equality in (A.51) comes from the independence of conditional outcomes. Note that $P\left(y_{-j}'|\boldsymbol{e^*_{-j}}, \mu_{-j}\right)$ is the *reputation* of seller $-j$, which represents a combination of *belief* and perceptions held by buyers (and therefore relevant for other sellers).

3. The dynamic reputation game is characterized by a *continuous action space* (the effort chosen by each seller-type is a continuous variable) and a continuous state space (the beliefs about a seller being of good type are also continuous in $[0,1]$). As it is well known in the game theoretic literature, the existence and uniqueness of an equilibrium under such features is quite different than under a *finite* action space (see for example Athey, (2001)). Therefore, the approximated solution needs to preserve such continuity in order to avoid one possible convergence problem. Within each iteration over $\boldsymbol{e}^*$, a new set of value functions (one for each seller and type) will be found. We approximate these (unknown) functions by means of linear approximation schemes, consisting of *basis* functions and approximation *nodes*; in other words, we will have $V_j\left(\boldsymbol{\mu};\theta\right) \approx \hat{V}_j\left(\boldsymbol{\mu};\theta,\{c\}\right)$ where $\{c\}$ is a vector of parameters.

The algorithm employed is a slightly modified version of the ideas developed in Ericson and Pakes, (1995) and Doraszelski and Pakes, (2007). We use a *successive approximation* strategy to the value functions of all players and types. Intuitively, we want to find (one of the possible) equilibrium in which the perceptions about effort, namely e^*, are indeed optimal strategies by sellers. The algorithm will find a *fixed point* of a mapping

$$\tilde{e}_j^\theta = F\left(\boldsymbol{e}^*, \cdot\right) \quad j = 1,2\ ;\ \theta = g, b$$

where $\boldsymbol{e}^* = \left\{e_j^{g*}, e_j^{b*}\right\}_{j=1,2}$ and $\tilde{e}_j^\theta$ is the optimal effort strategy for seller j and type θ.

The numerical algorithm used can be summarized as follow:

- *Step 0*: Choose an approximating method, m, and an objective function for the approximation. Here we use one spectral method (Chebyshev polynomials) and one finite-elements method (bi linear splines). The grid G_m (method m, with $m = \{cheb., spline\}$) used to approximate e_j^θ will be defined accordingly.

- *Step 1*: **Perceived effort iteration (PEI).** Choose an initial value for the perceived effort strategies, $e_{j,m}^{0*}\left(\mu_1,\mu_2;\theta\right),\ j = 1,2; \theta = \{g,b\}$ defined at the grid G_m for the approximating method m. Define a stopping criterion $\epsilon^{PEI} > 0$.

- *Step 2*: Given $e^{0*}_{j,m}$, and all the possible states (beliefs) today (μ_1, μ_2) compute the possible values of the beliefs tomorrow, (μ'_1, μ'_2), which depend only on the realizations of the transactions, y_1, y_2

$$\mu'_j = g\left(\mu_j, e^{0*}_{j,m}, y_j\right),\ j = 1, 2\ ; \theta \in \{g, b\}$$

where the law of motion $\mu' = g(\mu, \cdot)$ is derived from a Bayesian learning process.

- *Step 3*: **Value function iteration (VFI)**. Guess an initial value function $V^0_j(\boldsymbol{\mu};\theta)$ on the grid G_m for each seller j and type θ. Define a stopping criterion $\epsilon^{VFI} > 0$.

- *Step 4*: Projection step[8]. Use such guess and $\boldsymbol{\mu'}$ computed in *step 2* in order to approximate $V^0_j(\boldsymbol{\mu'};\theta)$ as

$$\hat{V}_j(\boldsymbol{\mu'};\theta, \{\boldsymbol{c_{ij}}\}) \equiv \sum_{k=0}^{n_1}\sum_{l=0}^{n_2} c_{kl} \cdot T_{i1}(\mu'_1)\, T_{j2}(\mu'_2)$$

where the coefficients $\{\boldsymbol{c_{kl}}\}$ are defined as

$$\{c\} = arg\min \int_{\mu_1 \times \mu_2} \left[V^0_j(\boldsymbol{\mu};\theta) - \hat{V}_j(\boldsymbol{\mu};\theta\{c\})\right]^2 d\mu_1 d\mu_2 \tag{A.52}$$

- *Step 5*: Maximization step. Compute $V^1_j(\cdot)$ by solving the linear programming problem

$$\begin{aligned} V^1_j(\mu_j, \mu_{-j};\theta) \;=\; & \max_{e^\theta_j} \Big\{ \boldsymbol{R}\left(\mu_j, \mu_{-j}, \boldsymbol{e^{0*}_m}\right) - c\left(e^\theta_j\right) + \\ & + \delta \sum_{y_j}\sum_{y_{-j}} p\left(y_j, y_{-j}|e^\theta_j, \boldsymbol{e^{0*}_m}, \mu_{-j}\right) \hat{V}_j\left(\mu'_j(y_j), \mu'_{-j}(y_{-j});\theta\right)\Big\} \qquad \text{(A.53)} \\ s.t.\ \ & e^\theta_j \in [\underline{e}, \bar{e}] \end{aligned}$$

- *Step 6*: If $\max\left\{|V^1_j - V^0_{j,}|\right\}_{j,\theta} < \epsilon^{VFI}$, stop and go to step 7. Else, update the guess by weighted function iteration: $V^0_j = \alpha V^1_j + (1-\alpha)V^0_j$, with $\alpha \in [0,1]$ and go to step 4.

[8]For the case of Chebyshev polynomials, we need to define the *number* of basis elements $\{T\}$, n_1 and n_2 (degree of approximating polynomials for both dimensions). Let $T_{i1}(h(\mu_1))$ and $T_{j2}(h(\mu_2))$ be the i-th (j-th) element of the n_1-th (n_2-th) degree Cheb. polynomial, where $h(\cdot) : [0,1] \to [-1,1]$. Given the properties that characterize the Chebyshev polynomial family, the coefficients $\{c_{ij}\}$ are relatively easy to compute (this includes the approximation of the integral in (A.52) using Gauss-Chebyshev quadrature, where we need to define the number of Chebyshev nodes $m_{\mu 1}$ and $m_{\mu 2}$, with $n_1 \leq m_{\mu 1}$ and $n_2 < m_{\mu 2}$) .
For the case of bi-linear splines, we choose a set of nodes (in the space of beliefs) $\{\mu_{1i}, \mu_{2j}\}_{i,j}$ which together with the basis $\{\psi\}_{i,j}$, they define the tensor approximation as

$$\hat{e}^\theta_j(\mu_1, \mu_2; \{c\}) \equiv \hat{e}(\mu_1, \mu_2, \theta; \{c\}) = \sum_{i=0}^{m_{\mu 1}}\sum_{j=0}^{m_{\mu 2}} c_{ij} \cdot \psi_i(\mu_1)\, \psi_j(\mu_2)$$

The basis (*tent* functions) are such that the coefficients take a simple form: $c_{ij} = V^\theta(\mu_{1i}, \mu_{2j})$. In addition, for $\mu_1 = \mu_{1i}$ and $\mu_2 = \mu_{2j}$ we have $\hat{e}^\theta(\mu_{1i}, \mu_{2j}) = e^\theta(\mu_{1i}, \mu_{2j})$ Note that, contrary to the strategy folowed for Chebyshev polynomials, we will have a *just-identified* system.

- *Step 7*: If $\max\left\{|\hat{e}^{\theta}_{j,m} - e^{0*}_{j,m}|\right\}_{j,\theta} < \epsilon^{PEI}$, where $\hat{e}^{\theta}_{j,m} = arg\max_{e^{\theta}_{j}}$ (A.53), stop. Else, update the guess by weighted function iteration: $e^{0*}_{j,m} = \alpha\hat{e}^{\theta}_{j,m} + (1-\alpha)e^{0*}_{j,m}$, with $\alpha \in [0,1]$ and go to step 2.

Some particularities of our model are worth noting. The fact that the stochastic vector of outcomes in our model, $\boldsymbol{y}$, (the outcome of transactions given effort decisions and types) takes discrete values (success or failure), implies that we don't need to employ numerical integration when computing expectations in *step 5*; this eliminates one source of numerical approximation error. However, we are still left with the errors arising from the approximation of $V_j(\cdot)$ in *step 4* (*fitting* step); in addition, we cannot avoid incurring in errors coming from an optimization or internal root finding algorithm required in *step 5*.

Given the assumptions regarding the strategies we allow for each seller, the numerical solution provided (time-invariant) effort decision rules.[9] Accuracy tests for the approximated solution and simulation are presented in the appendix.

Parametrization and Calibration

For the exercise to provide some relevant information, we need to impose discipline to the determination of parameter values.

Note that the dynamic problem for each seller implies that both the possible seller types as well as the number of buyers in each stage game, are fixed and the type space is given exogenously. We will then consider them as "parameters" of the game. The game, therefore, has two set of parameters: (i) parameters characterizing the payoff function of sellers (discount factor δ, the cost function $c(e)$, and the seller types g, b), (ii) parameters characterizing market conditions (number of buyers in the market, N)

Cost Function

The optimal decisions by sellers weight marginal costs of effort against future marginal increases in revenues, through the change in probabilities of a successful delivery. Although the characterisation (including dynamics) of the equilibrium will as presented in previous sections requires minimum assumptions on the cost function, the *quantitative* properties will depend directly on the parametrized cost function. The functional form chosen for the cost function is

$$c(e) = \begin{cases} \frac{\gamma_1}{(\gamma_2 - e)^2} + \gamma_3 \cdot e + \gamma_4 & \text{if } e \geq \underline{e} \\ 0 & \text{if } e \leq e \end{cases} \tag{A.54}$$

[9] To be precise, what we describe here is an "approximate" equilibria. As noted by **Kubler2005** such equilibriacan be far away from the "exact" equilibria. They provide sufficient conditions in order to ensure that such approximated equilibria is close to the exact one. Although such conditions are presented for competitive economies with heterogeneous agents, we suspect that similar concerns might arise in our dynamic game.

Such functional form is flexible enough in order to capture the minimum requirements. It presents 4 parameters: $\gamma_1, \gamma_2, \gamma_3, \gamma_4$; these will be calibrated "in equilibrium" matching 4 moments / conditions, out of which 2 correspond to: $c(\underline{e}) = 0$ and $c'(\underline{e}) = 0$.

The cost parameters, together with the discount factor β, the seller type values g, b and the number of buyers in the market N, will shape the effort decisions in equilibrium. These efforts e^* and the assumed market structure will determine the (random) evolution of reputation and revenues. Although at this point we still don't have access to the necessary on-line data in order to carry out a proper calibration / estimation, below we describe the main moments in the data analyzed in Cabral and Hortaçsu, (2010) as a reference.

Given our objective, namely quantifying the bias in the estimated elasticity of revenues η when a reduced form estimation is used, we will carry out two type of robustness checks: (i) different distances of seller types $g - b$ and (ii) different number of buyers in the market, N. Given the results of the characterization of the equilibrium in the 2-period game, changes in the values of these parameters have a significant impact on the properties of the equilibrium, and therefore might affect the size of the bias that arises in the estimation of η.

Cabral and Hortaçsu, 2010 analyzes a sample of 819 sellers of three different goods[10] and follow them at monthly intervals between October 24, 2002 and March 16, 2003. Among other features, they make emphasis on two moments observed in the dynamics of sales and feedback data[11]:

1. Conditional mean impact of the *first* negative feedback on sales growth rates.
2. Conditional mean impact of the *second and third* negative feedbacks on sales growth rates.
3. Mean number of transactions (time) until the first negative feedback.
4. Mean number of transactions (time) between the first and the second negative feedback.

The benchmark calibration is presented in table A.1.

[10] IBM ThinkPad T23, collectible coins and 1998 Holidays Teddy Beanie Babies.

[11] There is a fifth dimension of sellers behavior and strategies that is analyzed by Cabral20120, but which we do not consider here: the possibility for a seller to exit (change 0ne's identity or leave eBay). Although we do think that having the possibility to exit might change the importance of the reputation mechanism, the current version of our model does not include such possibility.

TABLE A.1: Parameter Values. Benchmark Model

Parameter	Value
Common discount factor δ	.9
Large gap in types, $g - b$	$g = .9\ \ b = .1$
Small gap in types, $g - b$	$g = .9\ \ b = .4$
Number of buyers, N	$N = 5$, $N = 10$ and $N = 50$
γ_1	1.2
γ_2	0.1
γ_3	0.058
γ_4	0.087
$\underline{e}$	0.5

Figures A.3 and A.4 present effort strategies (policy functions) and value functions respectively, in equilibrium, as a function of current seller reputations $\{\mu_j, \mu_{-j}\}$. The plotted solution corresponds to the case where the number of buyers in the market is $N = 10$ (an intermediate value), the possible types of sellers are either far apart ($g = 0.9$ and $b = 0.1$) or closer together ($g = 0.9$ and $b = 0.4$).[12] These graphs should be read in the same way as in the 2-periods case. Each curve in the left panels of figure A.3 plots a good-type seller (seller 1 in this case) optimal effort decisions as a function of her own's reputation, for different (fixed) values of her competitor's reputation. The impact of an increase in the competitors reputation has a similar effect to the 2 period case. However, the pattern of optimal decisions differs significficantly between the case when types are far apart ($g = 0.9$ and $b = 0.1$, top row) and the case when types are closer together ($g = 0.9$ and $b = 0.4$, bottom row). The former case is particularly different from the 2 periods case: note that now the optimal effort decision for a seller with high "ability" peaks at a reputation close to 0 and then becomes monotonically decreasing in her reputation; the optimal decision for a seller with low "ability" behaves in the opposite way, peaking very close to maximal reputation.

The reason for these behaviors can be hinted from the shape of the value functions for each type of seller, presented in figure A.4. A g-type seller knows that it is crucial to avoid getting close to zero reputation; once this reputation zone is avoided, chances are that she will be recognized by buyers and her competitors as a g-type seller in the long run. Due to this reason, the difference in net present value (NPV) between zero and near zero reputation is huge, and therefore the steepness of the value function in that region. On the other hand, and still considering the case ($g = 0.9$ and $b = 0.1$), a b-type sellers NPV around the lowest reputation does not change much. She is aware that it will be very tough for her to convince buyers in the long run that she is of g-type. The behavior of her NPV changes significantly as her reputation increases,

[12] We present effort and value functions for the other parameter values in the appendix.

since it becomes more likely that buyers end up believing she is a good seller. As her reputation increase, her NPV approaches at an even faster rate the NPV of a g-type, and therefore it becomes more profitable to invest in building the reputation.[13] Note that as we proved in proposition 6, the NPV (i.e. value functions) of both types of sellers are equal when their reputation is either 0 or 1.

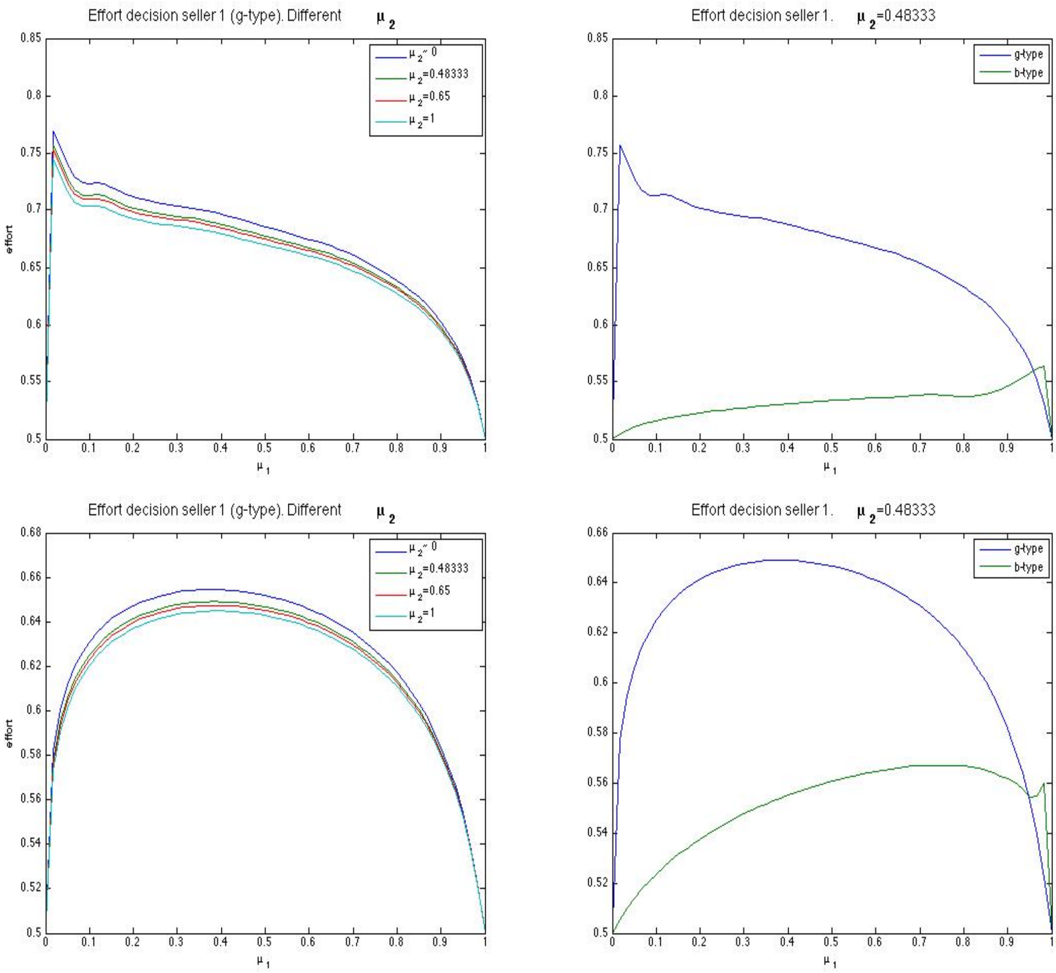


FIGURE A.3: Equilibrium effort strategies for $N = 10$: $g = 0.9$, $b = 0.1$ top row; $g = 0.9$, $b = 0.4$ bottom row

[13]Note that, although the value function of a bad seller near maximal reputation has a similar behavior as the value function of a good seller near minimal reputation, the effort of the bad seller is significantly smaller than the effort of the good seller. The reason is that in the optimality conditions, the benefit in an increase in reputation needs to be weighted by the change in the probability of a successful delivery, given by θ.

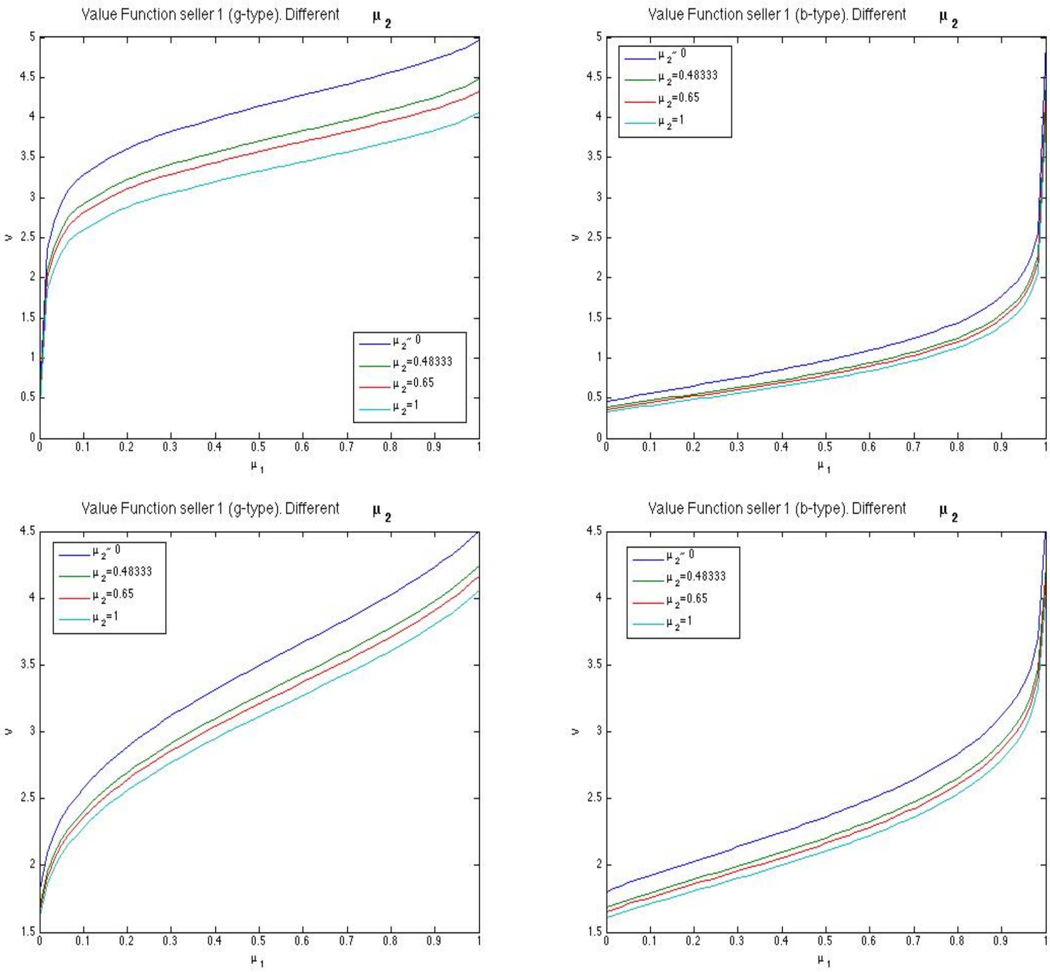


FIGURE A.4: Equilibrium Value Functions for $N = 10$: $g = 0.9$, $b = 0.1$ top row; $g = 0.9$, $b = 0.4$ bottom row

Figure A.5 presents the "true" reputation-elasticity of revenues generated in equilibrium. This is computed according to equation (A.44), using the approximated equilibrium solution to the game.[14]

[14]The accuracy of the approximation is discussed in appendix X.

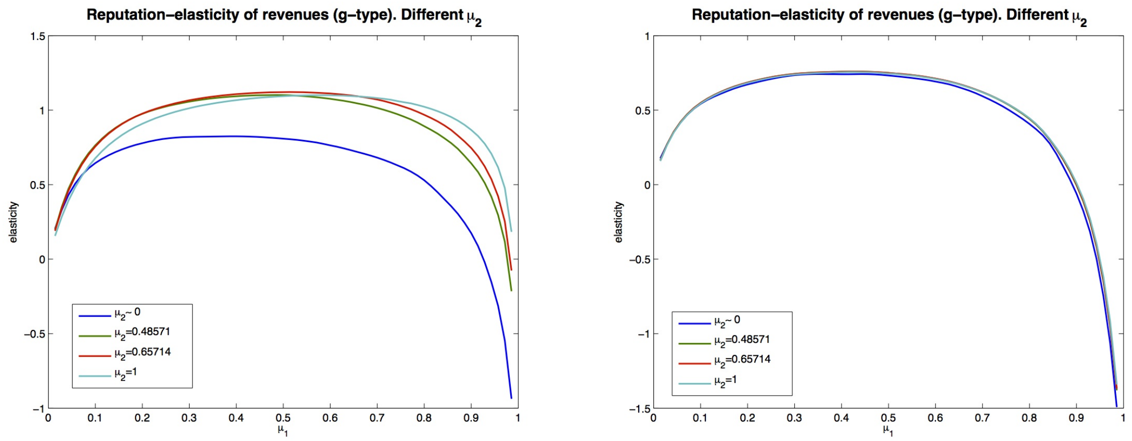


FIGURE A.5: Reputation-Elasticity of Revenues with $g = 0.9$, $b = 0.1$ and $N = 10$ (left figure) and $N = 50$(right)

Short-Run Value of Reputation: Are reduced-form estimates Biased?

We now answer the main question posed at the beginning of this section: are the estimates of the impact of reputation arising from reduced form econometric models still unbiased when strategic competition and reputation-building behavior are not accounted for? In other words, what information can a reduced form econometric estimation provide about the elasticity of revenues, as presented in figure A.5?

We proceed in the following way:

1. The numerical algorithm described above provides a solution to the infinite horizon game, characterized by optimal effort strategies $e_j^{*\theta}(\boldsymbol{\mu})$ for $\theta \in \{g, b\}$, $j = 1, 2$ a set of four value functions $\{V_j(\cdot; \theta)\}$, $j = 1, 2; \theta \in \{g, b\}$ and buyer's optimal selection and bidding strategies $\{\boldsymbol{s}_i(\boldsymbol{\mu}), b_i(\boldsymbol{\mu})\}$.

2. We then simulate M independent "panels" of (2) sellers competing for short lived buyers. These simulations will provide us with, among other variables, histories of seller revenues and reputations. More concretely, for a given set of parameter values (including the values of $\{g, b\}$), we simulate pairs of sellers under three different combinations of sellers types: (i) both sellers are of g-type; (ii) one seller is of g-type and one seller is of b-type; (iii) both sellers are of b-type. Obviously, the type realization of a seller is not known by buyers or the competitor. Then, starting from a set of initial beliefs $\{\mu_{1,0}, \mu_{2,0}\}$ we let each pair of sellers to compete during T transactions-periods. In our benchmark simulations, $M = 2000$ and $T = 200$.

3. We finally use such data in order to run reduced form regressions similar to the ones in Cabral and Hortaçsu, 2010 and **Resnik2006** and compare these with a "correctly specified" estimated regression.

Note that simulations are driven by three sources of randomness in each stage game, conditional on the current reputation values: first, buyers valuations are freshly drawn every period from the support $[0, 1]$ on which they are a priory independently and uniformly distributed. Secondly, which buyers bid in each auction is determined by each buyer's selection strategy $\boldsymbol{s}_i^*$, which is a vector of *probabilities* determining the odds of a buyer with an assigned private valuation of choosing each of the open auctions. Finally, after each auction has closed and the seller has decided how much effort to put into delivering the item $e_t^{*\theta}$, the outcome of the transaction (F or S) is determined randomly according to the technology $P(y = S|\theta, e^*) = \theta e^*$.

Generated Data: The dynamics of Reputation, Effort and Revenues

Figure A.6 presents a set of typical simulation paths for reputation, effort and (expected) stage revenues when the possible types of sellers are $g = 0.9$, $b = 0.1$. Each row in the figure presents the case in which two good-type sellers compete and the one in which a good and a bad seller compete. Figure A.7 presents a similar picture for the case when the possible types are $g = 0.9$, $b = 0.4$.

There are some features worth underlining. First, when the possible types are far apart, beliefs about the type of the seller converge much faster to the true distribution of types. Due to the low (high) intrinsic ability to succeed, a bad (good) seller faces a sequence of failures (successes) with high probability. This drive down (up) the beliefs and after a couple of transactions, there is convergence. In other words, the separation of types when sellers are either very professional or fake occurs very fast.

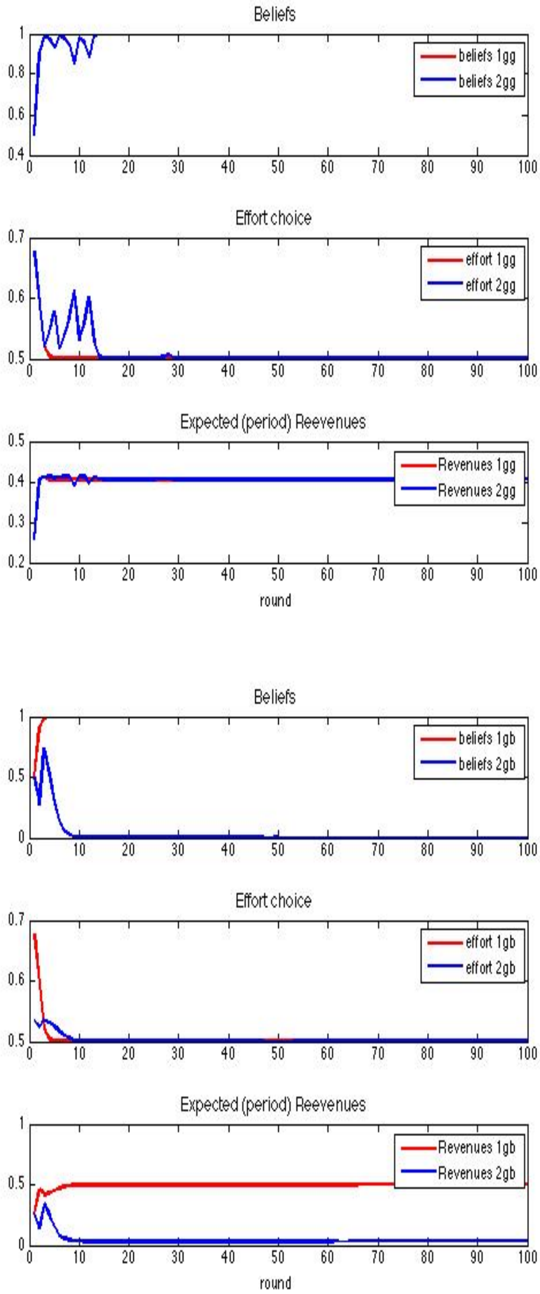


FIGURE A.6: Typical Simulations of Reputation, Efforts and Instantaneous Revenues Path. Two g-type sellers (left), one g-type and one b-type (right). Parameter values $N = 10$; $g = 0.9$, $b = 0.1$

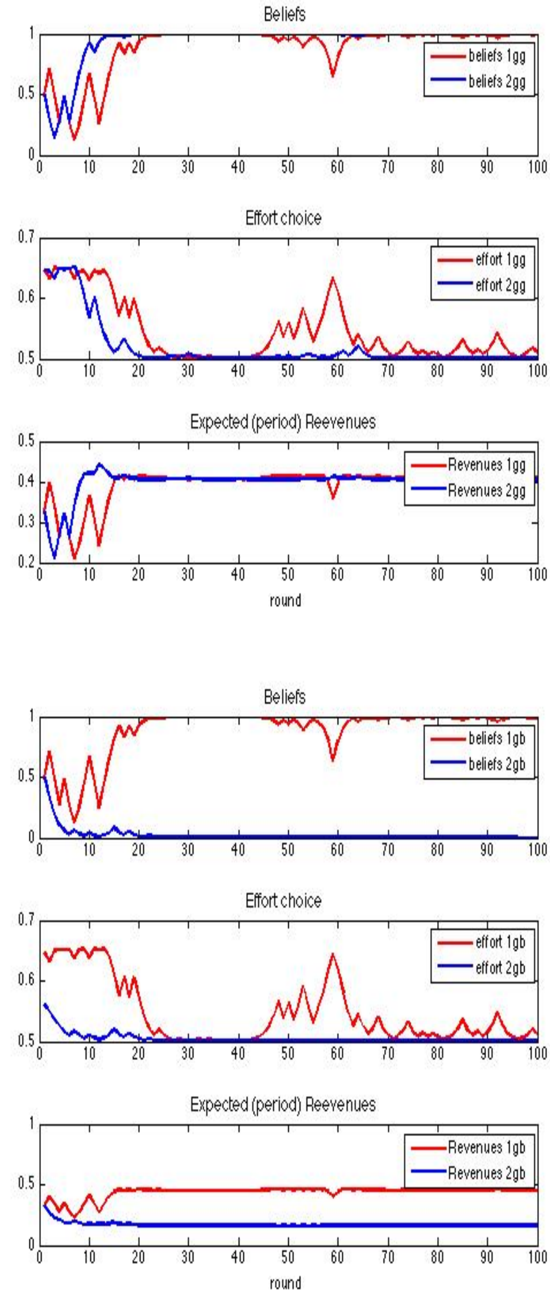


FIGURE A.7: Typical Simulations of Reputation, Efforts and Instantaneous Revenues Path. Two g-type sellers (left), one g-type and one b-type (right). Parameter values $N = 10$; $g = 0.9$, $b = 0.4$

TABLE A.2: Descriptive Statistics from Simulations

Reduced-Form Regressions

The simulation described above, in which M independent pairs of sellers compete for N short lived buyers during T periods, provide us with histories of reputation

$\left\{\left\{\mu^m_{j,t}, \mu^m_{-j,t}\right\}^T_{t=1,j=1,2}\right\}^M_{m=1}$, effort $\left\{\left\{e^m_{j,t}, e^m_{-j,t}\right\}^T_{t=1,j=1,2}\right\}^M_{m=1}$ and revenues $\left\{\left\{\boldsymbol{RR}^m_{j,t}, \boldsymbol{RR}^m_{-j,t}\right\}^T_{t=1,j=1,2}\right\}^M_{m}$

The final data used for the estimation, however, is a subset of the full sample: we discard observations for $t \geq t^*_j$, where t^* is the moment in which the reputation of seller j has converged to 0 or 1. The reason is simple: once the reputation has reached an absorbing state, then any variability in revenues will be independent of reputation. Including the observations of periods in which the reputation is maximal or minimal will therefore make the exercise less transparent.

We use the final dataset to estimate the following four *reduced-form* econometric specifications, including a version of equation (A.48) estimated by the mentioned previous empirical studies:

$$\log \boldsymbol{RR}^m_{j,t} = \beta^1_0 + f\left(\boldsymbol{\beta^1}, \log \mu^m_{j,t}\right) + \gamma^1 \log \mu^m_{-j,t} + \nu^1_{j,t} \tag{A.55}$$

$$\log \boldsymbol{RR}^m_{j,t} = \beta^2_0 + f\left(\boldsymbol{\beta^2}, \log \mu^m_{j,t}\right) + \nu^2_{j,t} \tag{A.56}$$

$$\log \boldsymbol{RR}^m_{j,t} = \beta^3_0 + \beta^3_1 \log \mu^m_{j,t} + \gamma^3 \log \mu^m_{-j,t} + \nu^3_{j,t} \tag{A.57}$$

$$\log \boldsymbol{RR}^m_{j,t} = \beta^4_0 + \beta^4_1 \log \mu^m_{j,t} + \nu^4_{j,t} \tag{A.58}$$

where $f\left(\boldsymbol{\beta}, \log \mu^m_{j,t}\right)$ is a non-linear function of $\log \mu^m_{j,t}$, though *linear in the parameters* $\boldsymbol{\beta}$.

Equation (A.55) approximates the "true" (model) expression for the (log) revenues of seller i in equilibrium, which includes a potentially non-linear dependence on her own reputation and a linear dependence on the competitors reputation. Equations (A.56)-(A.58) are the alternative (mis)-specifications considered, where, respectively, the dependence on the competitors reputation is omitted (eq. (A.56)), $f(\boldsymbol{\beta}, \cdot)$ is linear in reputation (eq. (A.57)) and where both mis-specifications are considered (eq. (A.58)). The fourth regression considered is the version considered in previous empirical studies such as Cabral and Hortaçsu, (2010).

Before presenting the results, some comments regarding the regressions and the simulated data are necessary. First, and as it is hinted from the dynamics of beliefs in figures A.6 and A.7, when the sellers are either very trustworthy or not trustworthy at all (i.e. $g = .9, b = .1$), the reputation of each seller converges to 0 or 1 quite fast. This implies that we are left with very few observations to estimate the regressions, since, as described above, we discard observations of the simulated time series for the periods after the reputation of a seller has reached an absorbing state. With a low number of observations, the estimated elasticity $\eta(\boldsymbol{\mu})$ is very imprecise. Second, the assumption about the distribution of buyer's valuations implies that when the number of buyers is small, the second highest valuation in the auction of any seller will be very volatile, independently of the reputation of the two sellers. With a very long time series, this wouldnt be a problem for the estimation of the elasticity; however, as mentioned before, the final simulated data sets are relatively short, which means that the variance of the estimated coefficients will be quite high. Third, given characteristics of the game andthe assumption about the outcome of the transaction tacking only two values (S or F), the simulated time series of reputation and revenues will be concentrated around a reduced number of values. This implies that estimating the entire (equilibrium) schedule of the elasticity $\eta(\boldsymbol{\mu})$ will be very difficult, since we will have few observations for certain values of the space of reputations.

Appendix B

Appendix Chapter 2

B.1 Data sources and definitions

B.1.1 National statistics

The aggregate time-series for the U.K. and the U.S. come from the Office for National Statistics (ONS) and the National Income and Product Accounts (NIPA), respectively. Personal consumption expenditure on non-durable goods and services expenditure, personal consumption expenditure on durable goods and disposable income are all seasonally adjusted at the source. The series are divided by total population to obtain per-capita values. The deflator used for the U.K. (U.S.) is the Retail Price Index excluding mortgage interest payments (Consumer Price Index).

B.1.2 Household survey data

For the U.K., we use the Living Costs and Food Survey (formerly known as Family Expenditure Survey) from 1975 to 2007 (1978 to 2007 when we use educational attainment for the probit regressions). For the U.S., we use the Consumer Expenditure Survey (interview section) from 1981 to 2007.

Household expenditure. *Non-durable goods and services*: includes food, alcohol, tobacco, fuel, light and power, clothing and footwear, personal goods and services, fares, leisure services, household services, non-durable household goods, motoring expenditures and leisure goods. Between 1982 and 1987, food at home in the CEX is adjusted following Aguiar and Bils, 2015. *Durable goods*: durable household goods, motor vehicles and durable leisure goods. This includes expenditure such as furniture and furnishings, electrical appliances and audio-visual equipment.

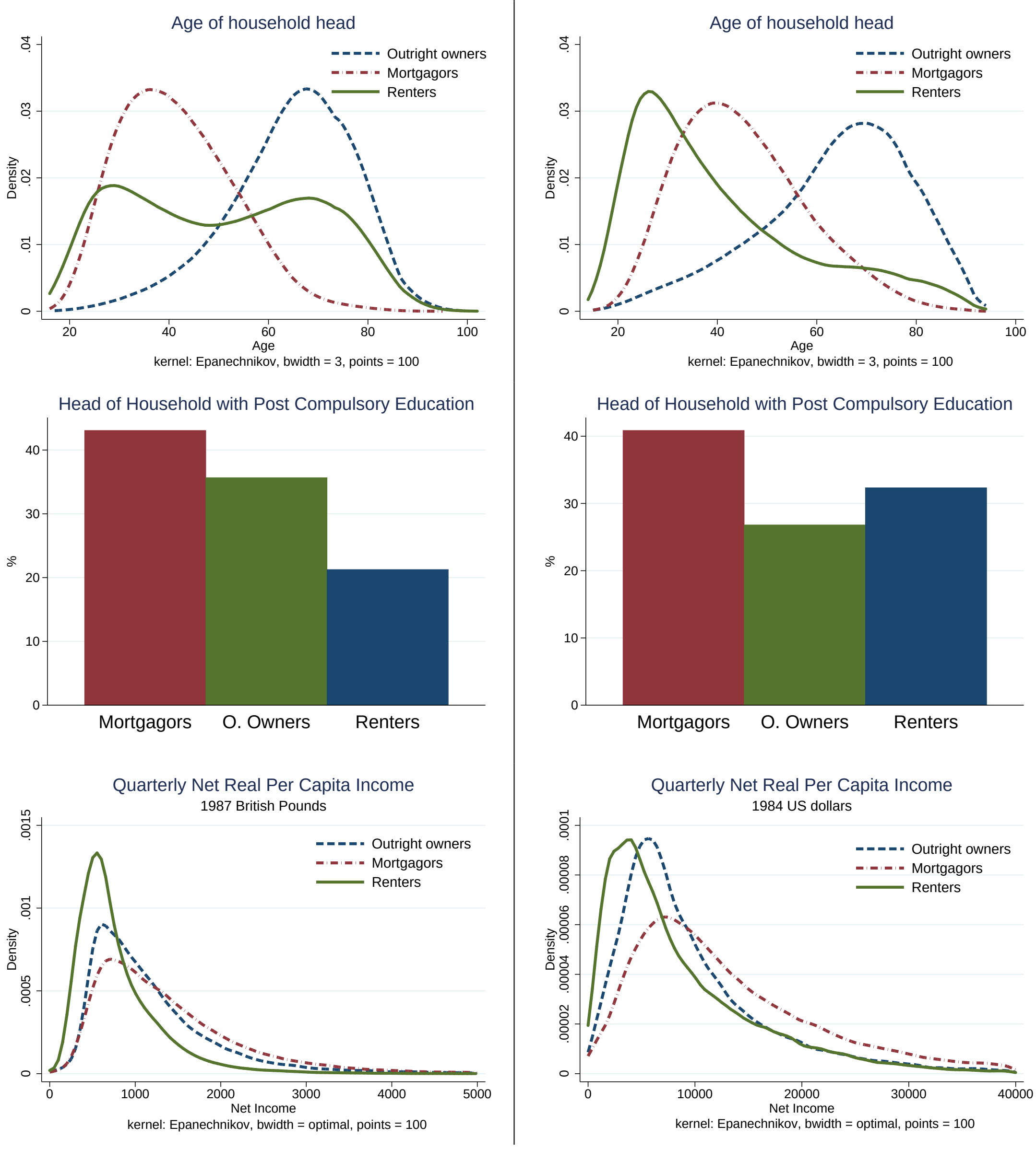
Mortgage payments and income. *Mortgage payments*: includes both interest payments and capital repayments (not available individually over the whole sample). *Net income*: sum of labor- and non-labor household income net of taxes paid.

Restrictions. We exclude households: (i) that do not report income, (ii) that report negative net income, (iii) that are in either the top or the bottom 1% of either the non-durable or the durable expenditure distributions of each housing tenure group at any quarter and (iv) whose head is either below 25 years old or above 74 years old. Finally, for the CEX, which features a short panel dimension, we only keep households that have not changed housing tenure status between interviews. The LCFS comprises repeated cross-sections and thus each household is observed only once. For comparability across countries and over time, all household variables in the LCFS for the U.K. (in the CEX for the U.S.) are deflated by the Retail Price Index excluding mortgage interest payments (Consumer Price Index) and divided by the household size to obtain real values in per-capita terms.

B.2 Housing tenure and demographics

Distribution of Demographic Characteristics Across Housing Tenure Groups

UK: 1975 - 2007 **US: 1981 - 2007**



Note: Top row: age of the household head at the time of interview; middle row: share of household heads who completed *more than* compulsory education; bottom row: real per capita income net of taxes. Left: U.K. (LCFS), 1975q1-2007q4. Right: U.S. (CEX), 1981q1-2007q4.

TABLE B.1: Mean Quarterly Household Expenditures and Income over the full sample, in 2007 US$

Panel A: United Kingdom

	non-durable expenditure	*durable expenditure*	*mortgage repayments*	*after-tax income*
Mortgagors	10,202	1,842	1,950	15,555
Outright Owners	10,030	1,625		13,617
Renters	6,807	750		9,310

Panel B: United States

	non-durable expenditure	*durables expenditure*	*mortgage repayments*	*after-tax income*
Mortgagors	14,767	2,470	2,352	20,500
Outright Owners	14,482	2,032		16,495
Renters	11,945	1,542		14,340

Note: Data for the UK comes from the Living Costs and Food Survey (LCFS) between 1975q1-2007q4; data for the US comes from the CEX between 1981q1-2007q4. The values in the table are average household level expenditures and income by cohort. In a given quarter these are constructed as a weighted average of all households within each cohort. Entries refer to sample averages converted into 2007 US dollars.

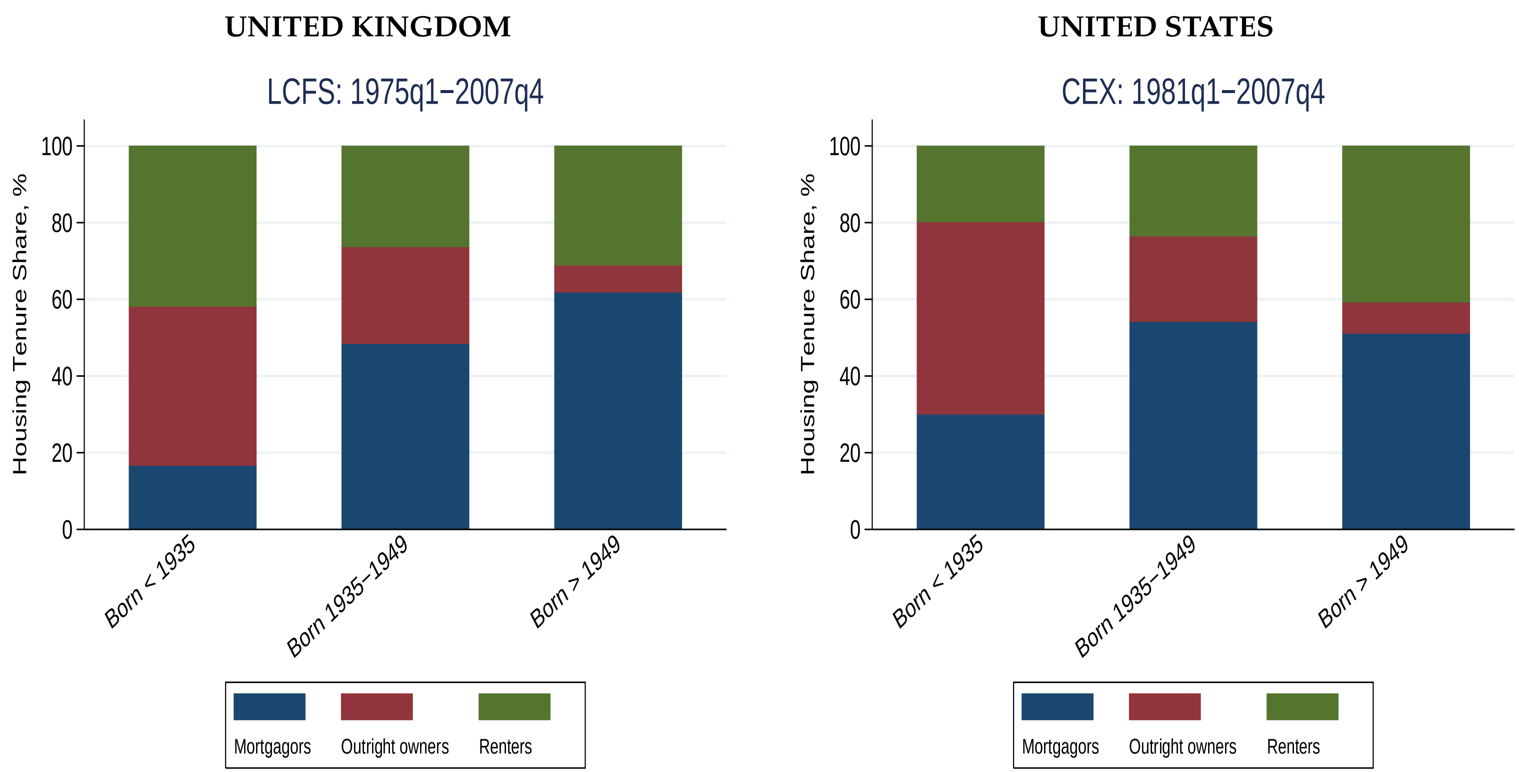


FIGURE B.1: The share of mortgagors, outright home owners and renters for different birth cohorts in the US and the UK. Birth cohorts are defined by the birth year of the household head. Three birth cohorts are considered: younger households, middle aged households and older households. The year cut-offs are chosen to ensure there are enough observations in each cohort group. The cut-offs also ensure that each birth cohort has approximately the same average age across the two countries.

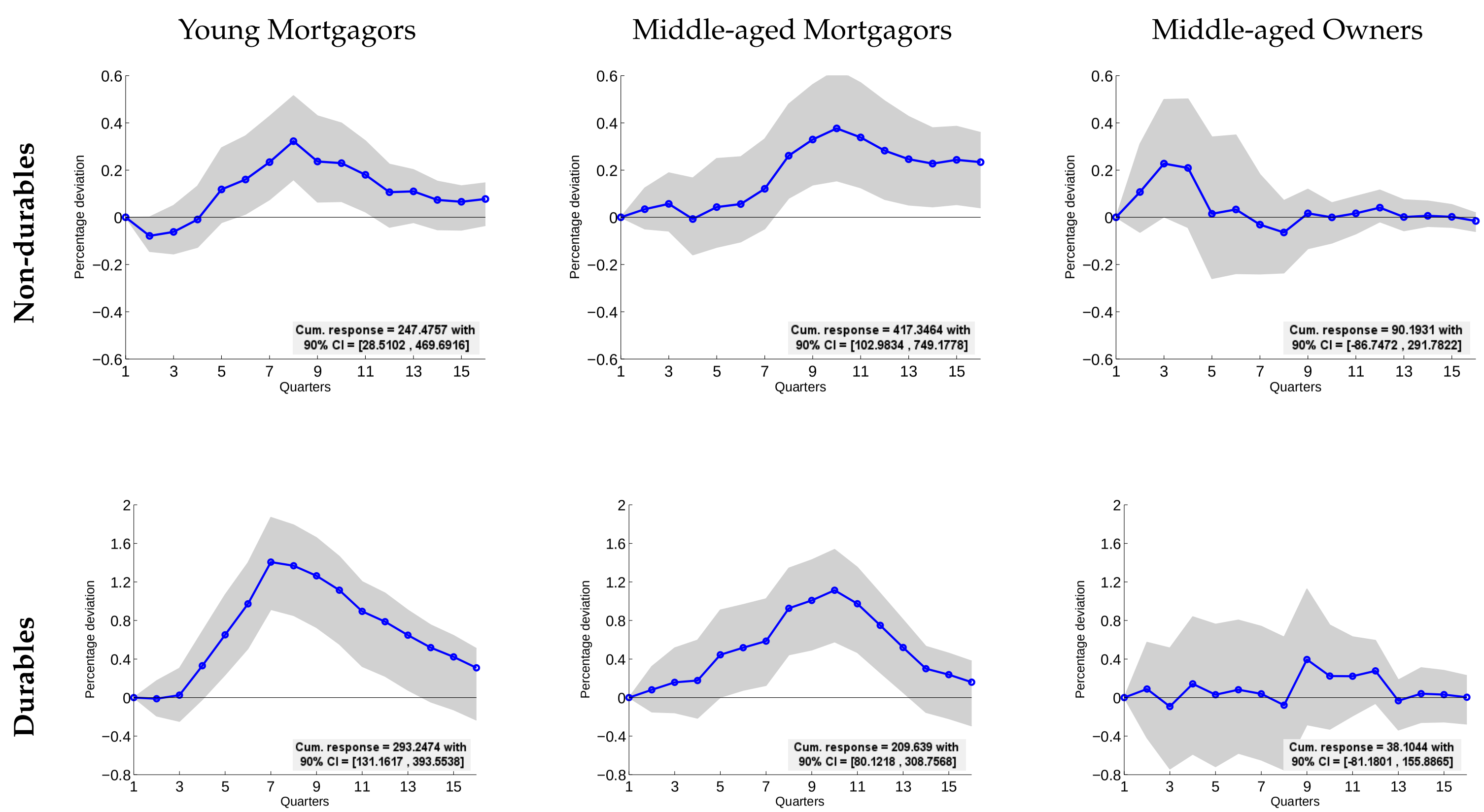


FIGURE B.2: Dynamic effects of a 25 basis point unanticipated interest rate cut on non-durable consumption (top) and durable expenditure (bottom) for mortgagors born after 1949 (left column), mortgagors born between 1935 and 1949 (middle column) and outright owners born before 1935 (right column). Grey areas are bootstrapped 90% confidence bands. U.K. data: FES/LCFS (1975-2007).

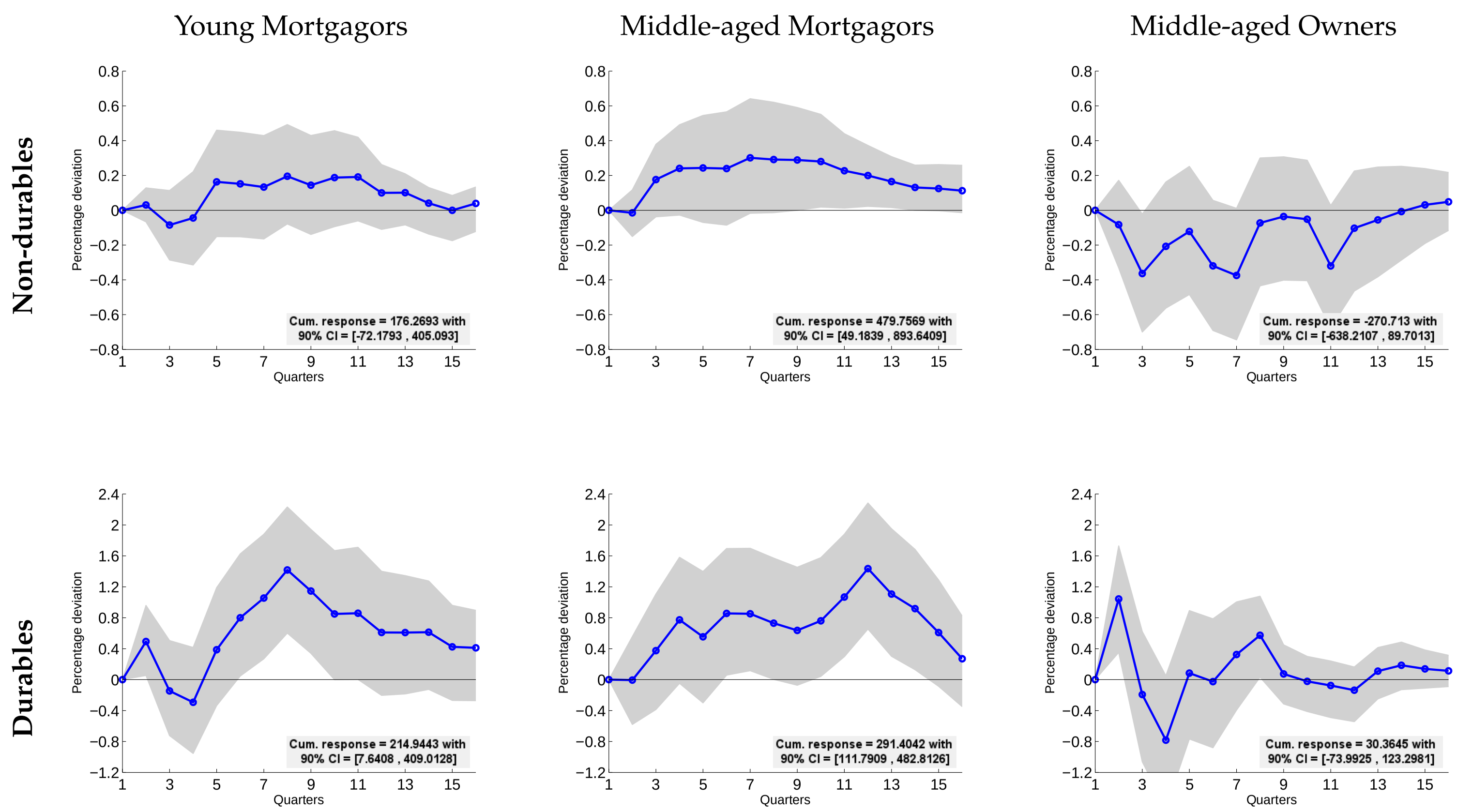


FIGURE B.3: Dynamic effects of a 25 basis point unanticipated interest rate cut on non-durable consumption (top) and durable expenditure (bottom) for mortgagors born after 1949 (left column), mortgagors born between 1935 and 1949 (middle column) and outright owners born before 1935 (right column). Grey areas are bootstrapped 90% confidence bands. U.S. data: CEX (1981-2007).

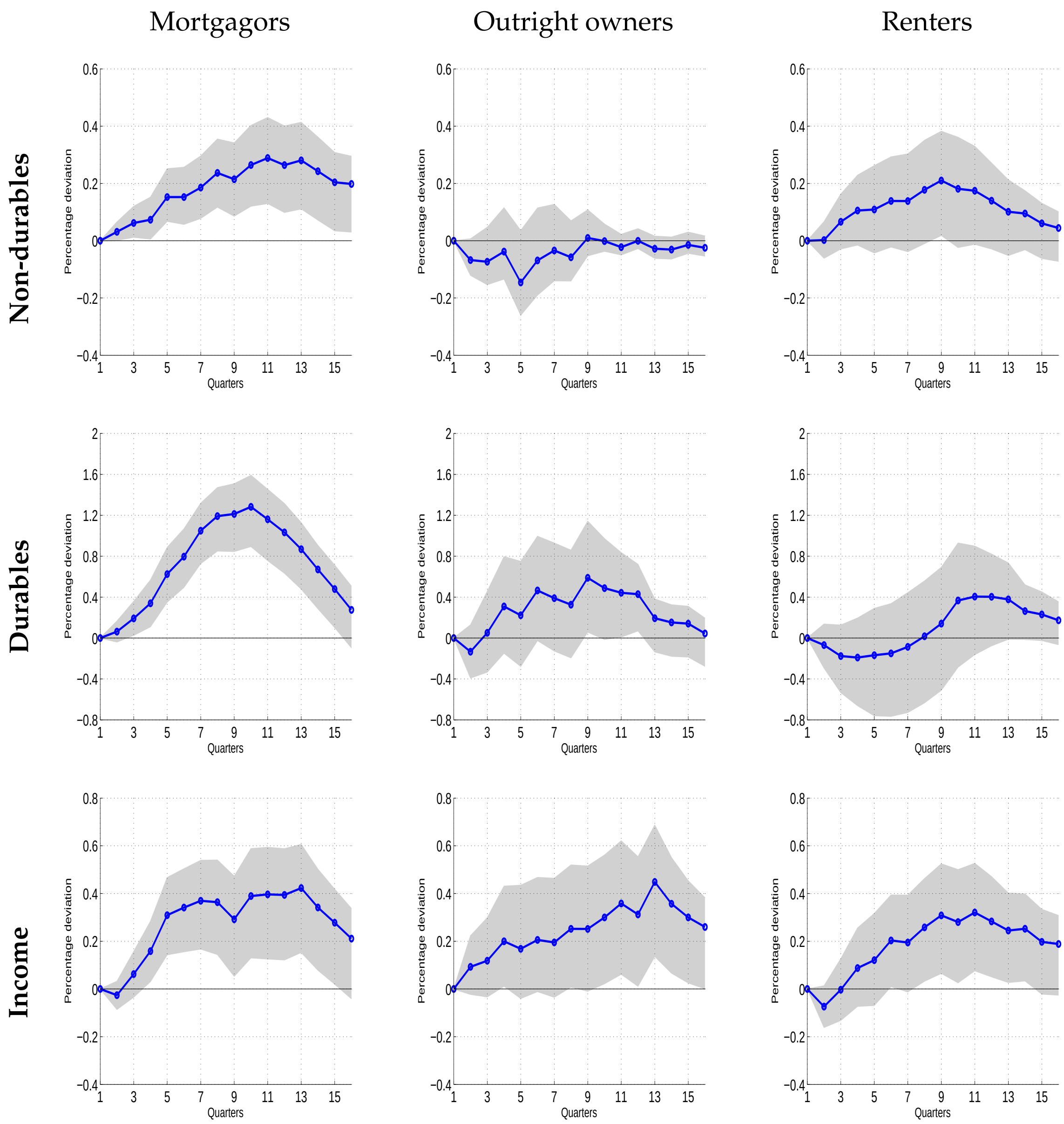


FIGURE B.4: Dynamic effects of a 25 basis point unanticipated interest rate cut on non-durable consumption (ND), durable expenditure (D) and income net of tax for households below age 65. U.K. data: FES/LCFS (1975-2007). Grey areas are bootstrapped 90% confidence bands.

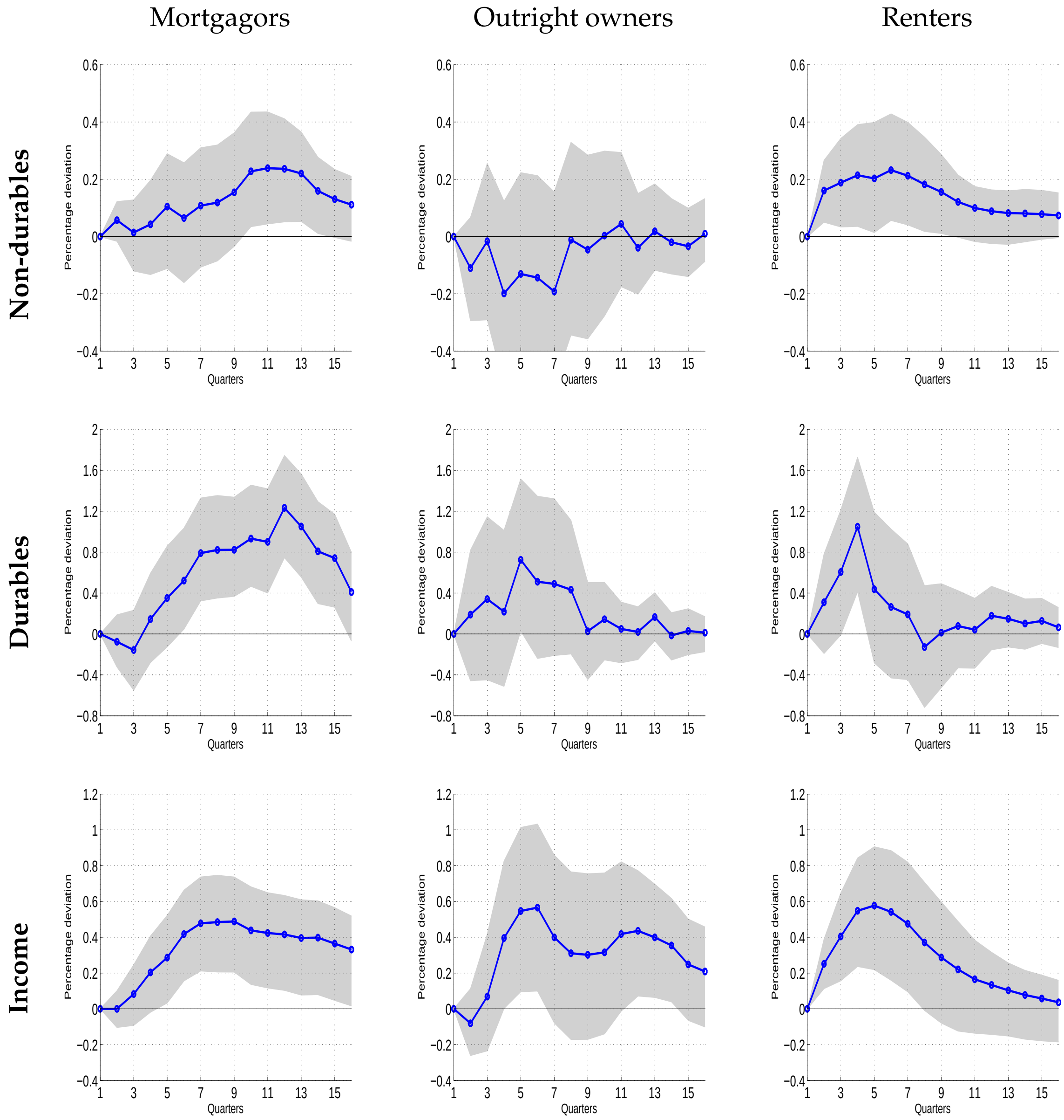


FIGURE B.5: Dynamic effects of a 25 basis point unanticipated interest rate cut on non-durable consumption (ND), durable expenditure (D) and income net of tax for households below age 65. U.S. data: CEX (1981-2007). Grey areas are bootstrapped 90% confidence bands.

B.3 WHTM by housing tenure

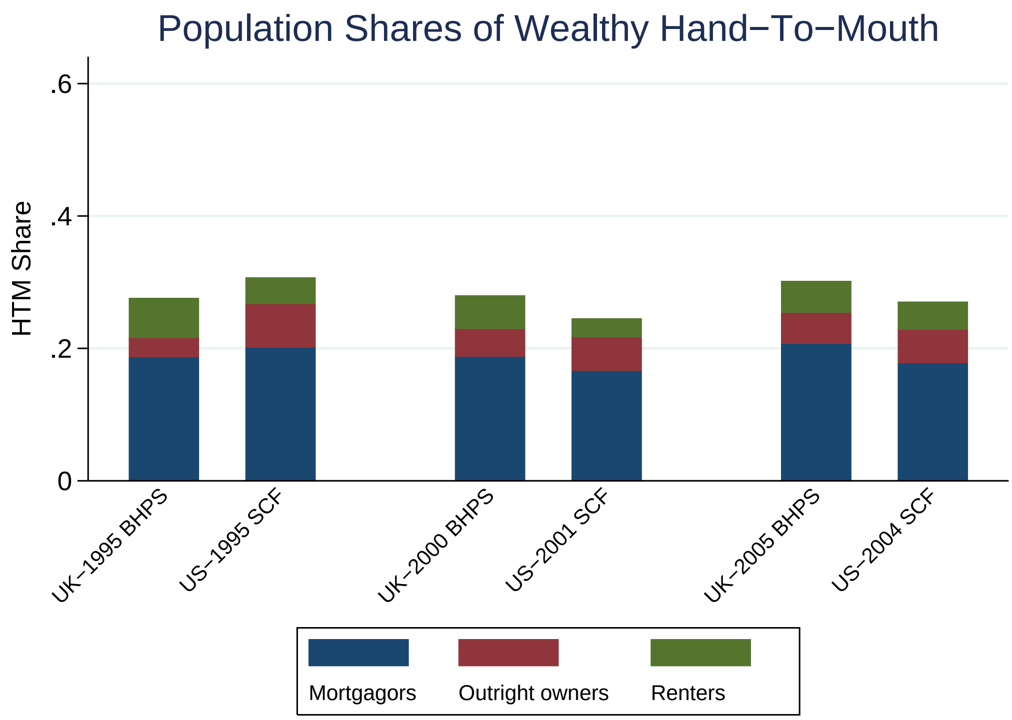


FIGURE B.6: Shares of Wealthy Hand-To-Mouth (WHTM) households in the population, by housing tenure group. U.K. (U.S.) data: 1995, 2000, 2005 waves of the British Household Panel Survey (Survey of Consumer Finances). A household is defined as WHTM if at any given point in time both (i) their net liquid wealth is less than half of their total monthly household labor income and (ii) their net illiquid wealth is positive. U.K. (U.S.) data: 1995, 2000, 2005 waves of the BHPS (SCF).

B.4 Compositional changes and renters

In this Appendix, we report the impulse responses for the shares of mortgagors, outright owners and renters as well as on non-durable consumption, durable expenditure and disposable income for renters in the U.K. and U.S..

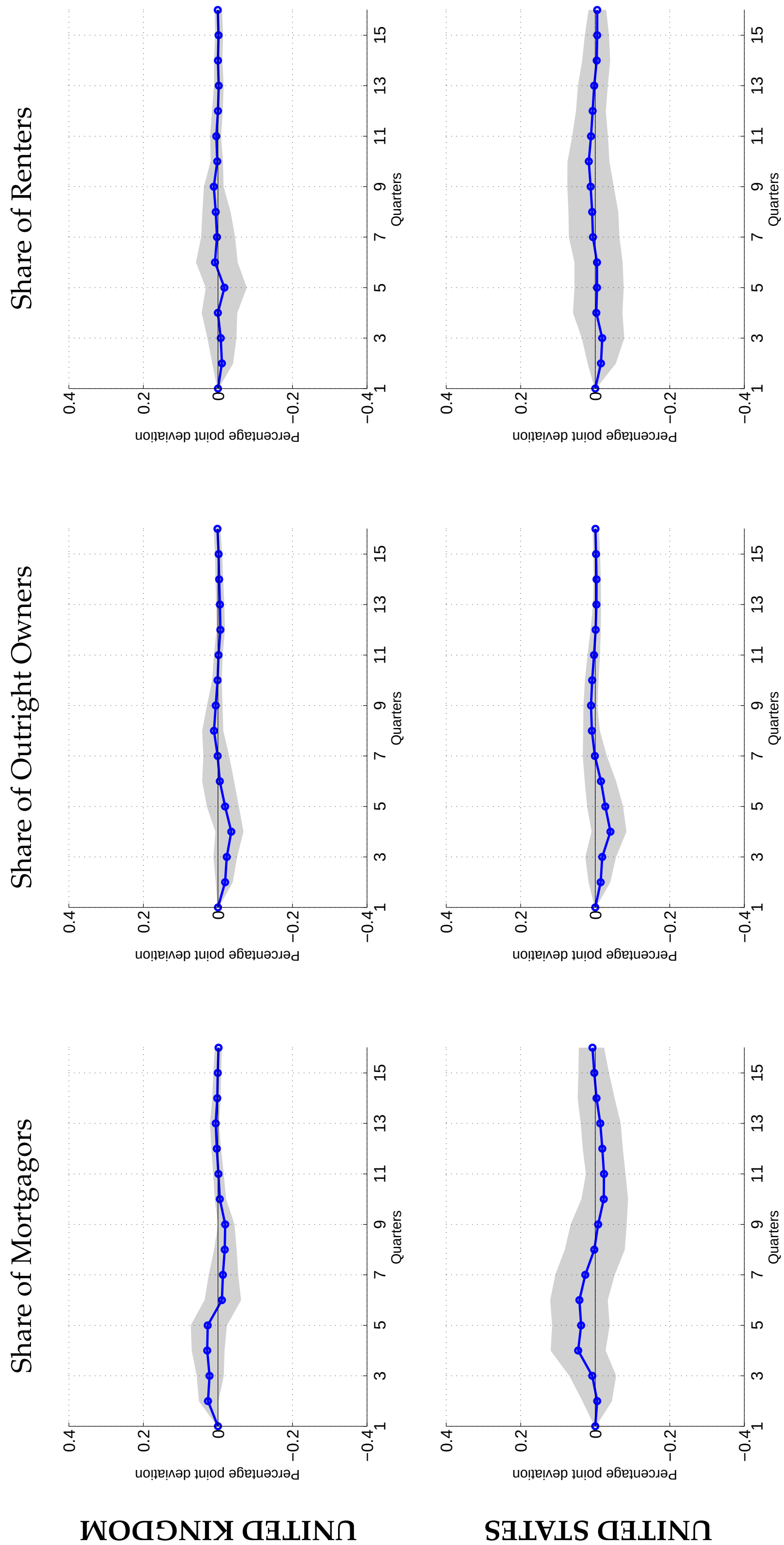


FIGURE B.7: Dynamic effects of a 25 basis point unanticipated interest rate cut on the share of mortgagors, outright owners and renters. Grey areas are bootstrapped 90% confidence bands. U.K. data: LCFS (1975-2007) in top row; U.S. data: CEX (1981-2007).

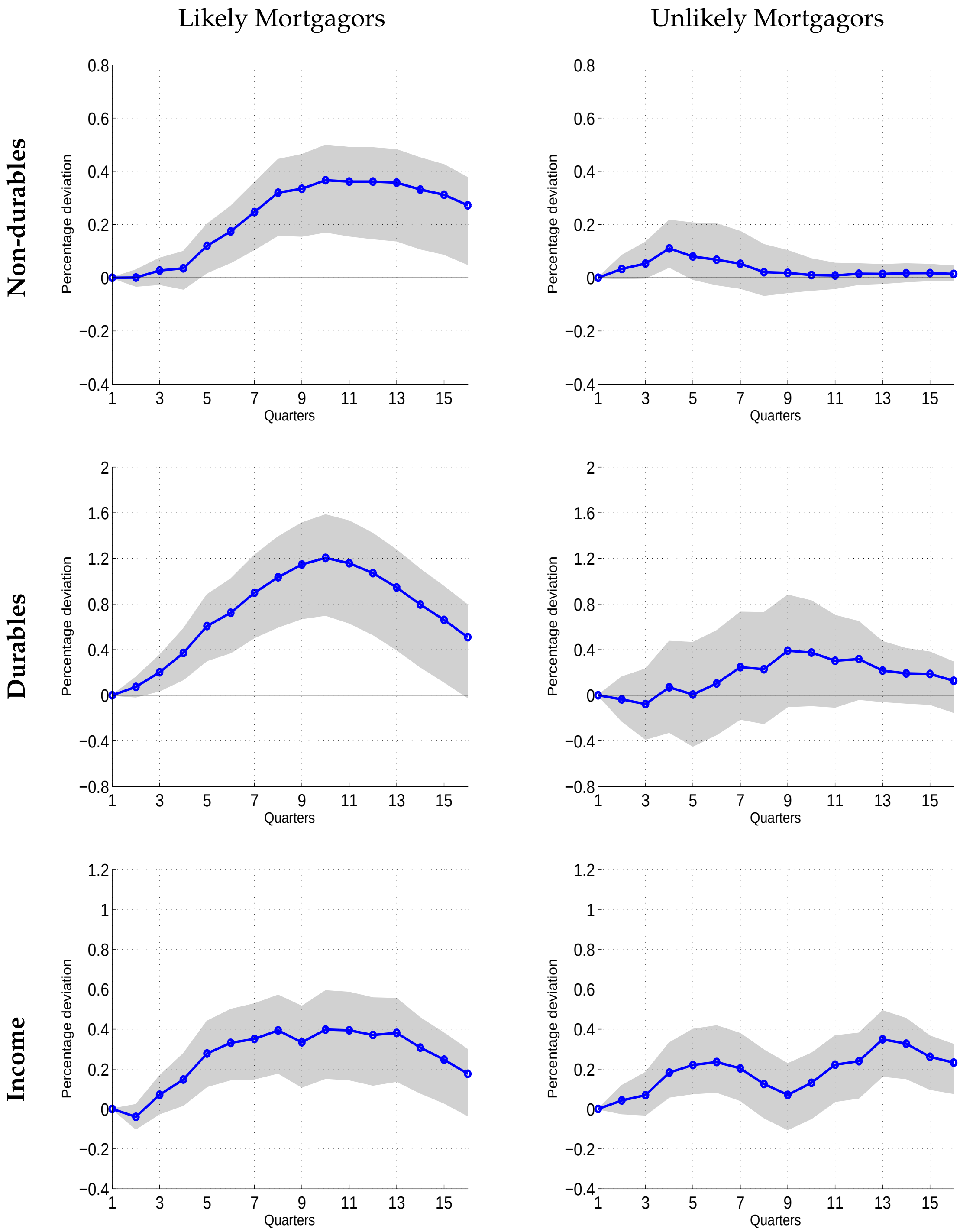


FIGURE B.8: UK results for non-durable, durable and income net of taxes for "likely" and "unlikely" mortgagors, excluding renters. Groups computed following Attanasio et al (2002) propensity score approach using a fixed probability threshold. Grey areas are bootstrapped 90% confidence bands.

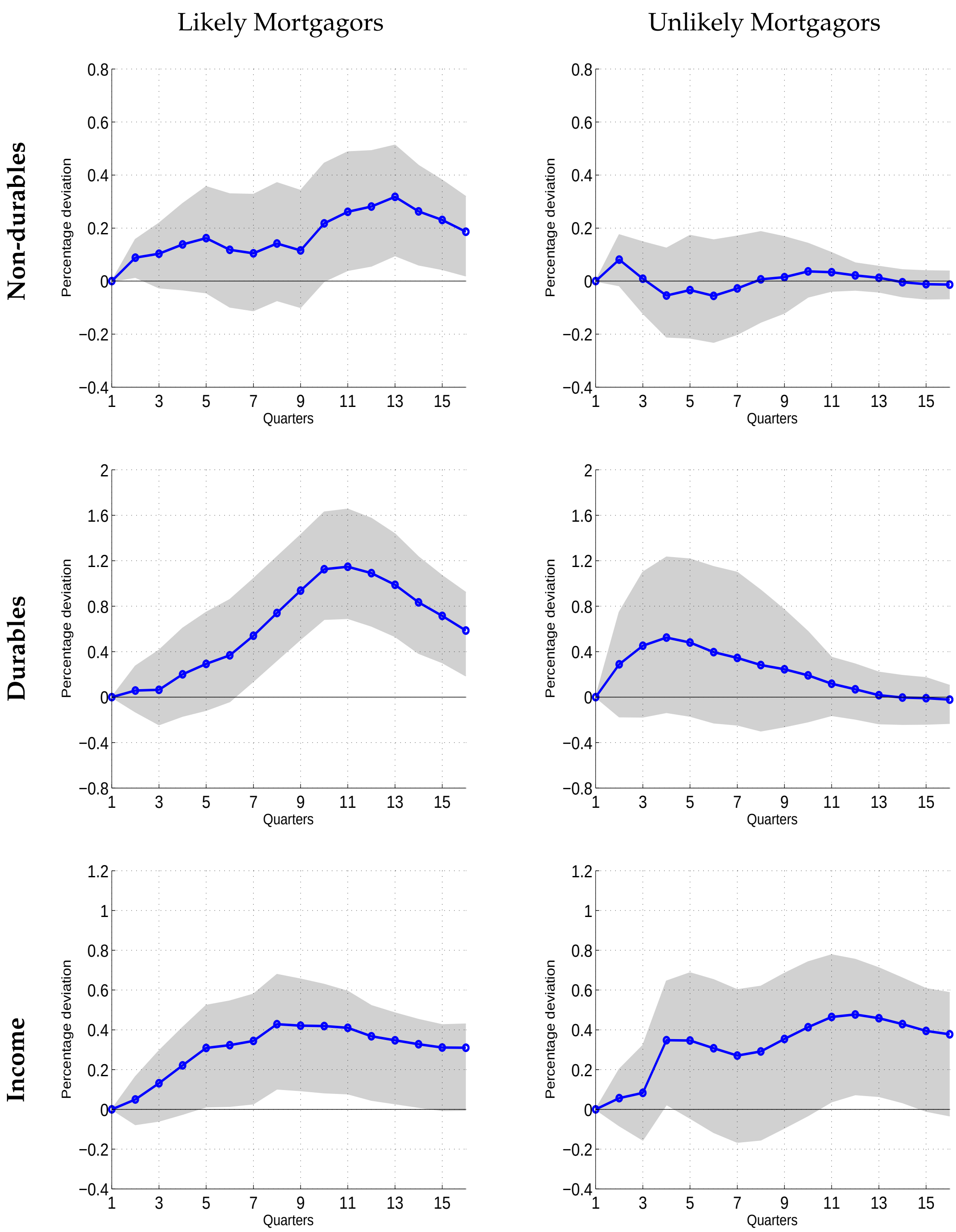


FIGURE B.9: US results for non-durable, durable and income net of taxes for "likely" and "unlikely" mortgagors, excluding renters. Groups computed following Attanasio et al (2002) propensity score approach using a fixed probability threshold. Grey areas are bootstrapped 90% confidence bands.

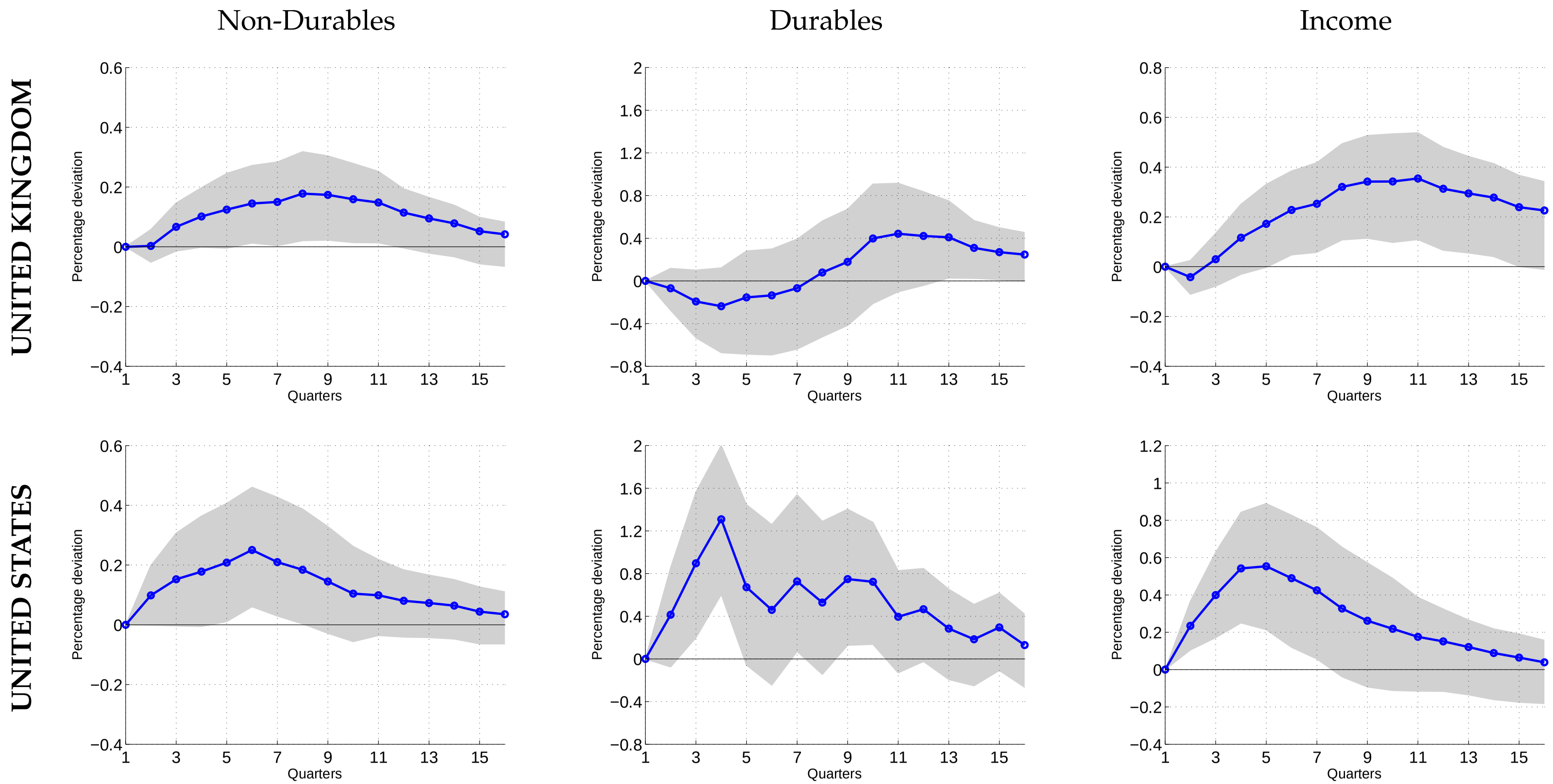


FIGURE B.10: Dynamic effects of a 25 basis point unanticipated interest rate cut on non-durable consumption (ND), durable expenditure (D) and income net of taxes for renters. Grey areas are bootstrapped 90% confidence bands. U.K. data: FES/LCFS (1975-2007) in top row. U.S. data: CEX (1981-2007).

B.5 The full model

B.5.1 Households

There are, ex-ante, two types of households: *patient households* (PH) and *impatient households* (IH). These are differentiated by their discount factors β^H and β^L respectively, with $0 < \beta^L < \beta^H < 1$.[1] They all derive utility from the consumption bundle x_t, housing stock and/or services $\tilde{h}_t$ and dis-utility from labor L_t:

$$E_0\left[\sum_{t=0}^{\infty}\left(\beta^i\right)^t\left(\frac{x_t^{1-\sigma}}{1-\sigma}+j\log\tilde{h}_t-\frac{L_t^{\eta}}{\eta}\right)\right]$$

where $\sigma > 0$ is a curvature parameter, j is a housing demand parameter, and $\eta > 0$ is related to the Frisch elasticity of labor supply. The consumption bundle x_t is defined as

$$x_t \equiv C_t^{\theta}V_t^{1-\theta} - \mu C_{t-1}^{\theta}V_{t-1}^{1-\theta} \tag{B.1}$$

with C_t and V_t being non-durable consumption and the stock of durables, receptively; $\theta \in [0,1]$ is a share parameter and $\mu \in [0,1)$ captures habit persistence. The stock of durables for a household evolves according to the following law of motion:

$$V_{t+1} = \left(1-\Phi\left(\frac{D_t}{D_{t-1}}\right)\right)D_t + (1-\delta)V_t \tag{B.2}$$

where D_t denotes purchases of new durables, $\Phi\left(\frac{D_t}{D_{t-1}}\right) = \frac{\phi_d}{2}\left(\frac{D_t}{D_{t-1}}\right)^2$ captures the costs of adjusting durables, and δ is the rate of depreciation of consumer durables. There is a fixed stock of houses H, which are sold and bought at a price q_t^h.

B.5.2 Assets

Mortgages

Households can borrow/save through a multi-period (long term) instrument, which we refer to as bond or mortgage. One unit of debt issued at t pays, starting in $t+1$, the sequence of nominal installments $1, \rho, \rho^2, \ldots$, which decay at a rate $\rho \in [0,1]$. Therefore, if M_t units of debt are issued at t, the mortgage installment at $t+j$ is given by

$$pay_{t,t+j} = \rho^{j-1}M_t \quad j \geq 1 \tag{B.3}$$

[1] We assume that impatient (patient) households can trade a complete set of arrow securities with other impatient (patient) households, but not with the patient (impatient) households. This implies that idiosyncratic risk can be perfectly shared within households of the same type, but other risks cannot be insured with households of a different type.

The *total amount of installment payments* due at t is then given by

$$\begin{aligned} B_t &= \sum_{j=1}^{t} pay_{t-j,t} \\ &= M_{t-1} + \rho B_{t-1} \end{aligned} \tag{B.4}$$

Given this notation, the *value of the stock* of debt at the beginning of each period is given by

$$B_{t+1}^{stock} = \left(M_t + \rho M_{t-1} + \rho^2 M_{t-2} + ... + \rho^t M_0\right) \cdot S_t = B_{t+1} \cdot S_t$$

where S_t is the time-t price of one unit of debt. The value of the *real* stock is given by

$$b_{t+1}^{stock} \equiv \frac{B_{t+1}^{stock}}{P_{c,t}} = b_{t+1} S_t = \left(m_t + \rho \frac{b_t}{\pi_{c,t}}\right) S_t \tag{B.5}$$

where $m_t \equiv \frac{M_t}{P_{c,t}}$, $b_t \equiv \frac{B_t}{P_{c,t}}$ and $\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}$.

One-period bonds

PH can also save through a nominal one-period bond, traded in zero net supply. One unit of such bond can be bought at a price of one, and earns a nominal return of R_t.

B.5.3 Production of durables and non-durables

Following Monacelli, 2009, we model two sectors, producing durable investment goods D and non durable goods C using labor as the only input. In each sector, there are competitive final good producers and monopolistically competitive producers of intermediate varieties, facing the same cost of adjusting prices following Rotemberg, 1982.

In the *symmetric equilibrium* where intermediate producers use the same amount of labor, it is possible to obtain an expression for the evolution of prices in each sector (Phillips curves)

$$\hat{\pi}_{D,t} = \beta^H E_t\left(\hat{\pi}_{D,t+1}\right) + \left(\frac{\epsilon_D - 1}{\vartheta_D}\right) \hat{mc}_{D,t} \tag{B.6}$$

$$\hat{\pi}_{c,t} = \beta^H E_t\left(\hat{\pi}_{c,t+1}\right) + \left(\frac{\epsilon_C - 1}{\vartheta_C}\right) \hat{mc}_{C,t} \tag{B.7}$$

where " ˆ " variables denote deviation form a zero-inflation steady state, and $\pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}}$ is the gross inflation rate in sector j.

Monetary policy

We assume that monetary policy is conducted through the short term (one-period) rate R_t, following a Taylor rule:

$$R_t = (R_{t-1})^{r_R} \left(\pi_{t-1}^{1+r_\pi} \left(\frac{Y_{t-1}}{Y} \right)^{r_Y} \bar{rr} \right)^{1-r_R} \epsilon_{R,t}$$

where $\pi_t \equiv \pi_{c,t}^{\alpha_\pi} \pi_{D,t}^{1-\alpha_\pi}$ is a composite inflation index, $Y_t = Y_{C,t}^{\alpha_y} Y_{D,t}^{1-\alpha_y}$ is a composite output index and $\bar{rr}, Y$ are steady state real rate and output.

B.5.4 The model with an exogenous credit limit

We first consider, along the lines of Eggertsson and Krugman, 2012, the case where households face an exogenous[2] credit limit of the form

$$S_t b_{t+1} \leq E_t \left(\bar{\Omega} \frac{\pi_{t+1}}{R_t} \right) \tag{B.8}$$

If we define positive values of b_t as debt (negative values are savings), then the real budget constraint (in terms of the non-durable consumption good) for the impatient household reads

$$C_t + q_t^h \Delta h_t + q_t^d D_t + \frac{b_t}{\pi_{c,t}} = w_t L_t + S_t m_t \tag{B.9}$$

and for the patient household:

$$C_t^{'} + q_t^h \Delta h_t^{'} + q_t^d D_t^{'} - \frac{b_t^{'}}{\pi_{c,t}} = w_t L_t^{'} - S_t m_t^{'} + \Pi_{C,t} + \Pi_{D,c} \tag{B.10}$$

where patient household variables are denoted by "$'$".

The problem for the impatient household is to maximize the following expression:

$$W_{IH} = \max_{\{C_t, D_t, V_{t+1}, h_t, L_t, b_{t+1}, m_t\}} E_0 \sum_{t=0}^{\infty} \left(\beta^L\right)^t u\left(x_t, h_t, L_t\right)$$

subject to the budget constraint (B.63), the credit constraint (B.8), and the law of motion for the durable stock and the outstanding debt, (B.2), (B.4). We also assume a transversality (No-Ponzi games) condition.

Denote by λ_t the multiplier on the budget constraint; $\lambda_t q_t^v$ the multiplier on the law of motion for the durable stock, $\lambda_{M,t}$ the multiplier on the law of motion for mortgage

[2] One could also think of this as a borrowing limit tied to the steady state value of collateral.

repayments, and $\lambda_{CC,t}$ the multiplier on the credit constraint. The optimality conditions for $C_t, D_t, V_{t+1}, L_t, h_t$ as well as the Euler equation, can then be written respectively as

$$\lambda_t = \theta\left(\frac{C_t}{V_t}\right)^{\theta-1}\left(x_t^{-\sigma} - \mu\beta^L E_t\left(x_{t+1}^{-\sigma}\right)\right) \tag{B.11}$$

$$\begin{aligned} q_t^d &= q_t^v\left(1 - \Phi\left(\frac{D_t}{D_{t-1}}\right) - \frac{\partial\Phi\left(\frac{D_t}{D_{t-1}}\right)}{\partial D_t}D_t\right) + \\ &+\beta^L E_t\left(\frac{\lambda_{t+1}q_{t+1}^v}{\lambda_t}\frac{\partial\Phi\left(\frac{D_{t+1}}{D_t}\right)}{\partial D_t}D_{t+1}\right) \end{aligned} \tag{B.12}$$

$$\lambda_t q_t^v = \beta^L E_t\left(\lambda_{t+1}\left(\frac{1-\theta}{\theta}\frac{C_{t+1}}{V_{t+1}} + (1-\delta)q_{t+1}^v\right)\right) \tag{B.13}$$

$$L_t^{\eta-1} = \lambda_t w_t \tag{B.14}$$

$$\lambda_t q_t^h\left(1+\phi_h\frac{\Delta h_t}{h_{t-1}}\right) = \frac{j}{\tilde{h}_t} + E_t\left(\beta^L\lambda_{t+1}q_{t+1}^h\left(1+\phi_h\frac{\Delta h_{t+1}}{h_t}\right)\right) \tag{B.15}$$

$$\lambda_t = \beta^L E_t\left(\lambda_{t+1}\frac{1+\rho S_{t+1}}{S_t\pi_{t+1}}\right) + \lambda_{CC,t}R_t \tag{B.16}$$

Condition (B.11) equates the shadow value of relaxing the budget constraint to the marginal utility of non-durable consumption, which is a function of habits μ. Conditions (B.12) and (B.13) are standard, and describe the expenditure and stock decisions for durables.[3] Equation (B.14) is the usual intratemporal condition equating the marginal rate of substitution between non-durable consumption and labor to the real wage. This condition is important to understand how a constrained household adjusts hours worked in order to compensate for being borrowing constrained. Equation (B.15) relates the shadow price of consumption to the marginal service value of housing in the current period, and its expected resale value in the next period. Note that in this version of the model, the only difference between D and housing is how they enter the utility of households. Finally, (B.16) is the modified Euler equation, which involves the (expected) one-period *holding return* on the long term mortgage $\frac{1+\rho S_{t+1}}{S_t\pi_{t+1}}$.

The problem for the PH is similar, but only facing constraints (B.64) and (B.2), (B.4). The main difference arises in the Euler equation which, for the patient household, reads

$$\lambda_t^{'} = \beta^H E_t\left(\lambda_{t+1}^{'}\frac{1+\rho S_{t+1}}{S_t\pi_{t+1}}\right) \tag{B.17}$$

[3] To see things clearly, assume for the moment no adjustment costs, i.e. $\Phi = \Phi^{'} = 0$. Then these two conditions can be combined into a standard optimality condition

$$q_t^d = \beta^L E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\left(U_{V,t} + (1-\delta)q_{t+1}^d\right)\right)$$

No-arbitrage pricing

The price S_t of the mortgage can then be written as

$$S_t = E\left(\sum_{j=1}^{\infty} Q_{t,t+j}\rho^{j-1}\right) = \frac{1}{R_t} + \rho E\left(Q_{t,t+1}S_{t+1}\right) \tag{B.18}$$

where $Q_{t,t+j} = \left(\beta^H\right)^j \frac{\lambda_{t+j}}{\lambda_t}$ for $j \geq 1$ is the stochastic discount factor (SDF) of the patient household, between t and $t+j$. This second equality assumes, crucially, that there is *no arbitrage* when pricing. This means that a condition needs to be satisfied between the return of the long term debt and an implicit one-period bond.[4]

We can define the (Macaulay) duration of this contract as

$$D_t(\rho) = \sum_{j=1}^{\infty}\left(Q_{t,t+j}\cdot j\frac{\rho^{j-1}}{S_t}\right) \tag{B.19}$$

In a zero inflation steady state, equation (B.17) implies $Q_{t,t+j} = \frac{1}{R^j} = \left(\beta^H\right)^j$ and therefore the mortgage price in steady state is

$$S = \frac{\beta^H}{1-\beta^H\rho}$$

while the steady state duration is equal to

$$D(\rho) = \frac{1}{1-\beta^H\rho} \tag{B.20}$$

Market Clearing

Goods, labour, housing and debt markets need to clear. In other words

$$\begin{aligned}
Y_{C,t} &= \omega_{IH}\left(C_t + \xi_{h,t}\right) + \omega_{PH}\left(C_t^{'} + \xi_{h,t}^{'}\right) + \frac{\vartheta_C}{2}\left(\pi_{C,t}-1\right)^2 Y_{Ct} && \text{(B.21)}\\
Y_{D,t} &= \omega_{IH}D_t + \omega_{PH}D_t^{'} + \frac{\vartheta_D}{2}\left(\pi_{D,t}-1\right)^2 Y_{Dt} && \text{(B.22)}\\
L_{C,t} + L_{D,t} &= \omega_{IH}L_t + \omega_{PH}L_t^{'} && \text{(B.23)}\\
H &= \omega_{IH}h_t + \omega_{PH}h_t^{'} && \text{(B.24)}\\
0 &= \omega_{IH}b_t + \omega_{PH}b_t^{'} && \text{(B.25)}
\end{aligned}$$

[4] Up to a first order approximation, the key no-arbitrage pricing condition can be stated as

$$E_t\left(\frac{R_t}{\pi_{t+1}}\right) = E_t\left(\frac{1+\rho S_{t+1}}{S_t\pi_{t+1}}\right)$$

Alternatively, we could also have assumed markets for two types of assets: a one-period bond, and a long-term mortgage, with households being able to use the short term bond only as a *saving* instrument which pays a gross *nominal* interest rate R_t^1. On the other hand, the mortgage market can be used *both* for saving and borrowing.

Competitive Equilibrium

A *competitive equilibrium* in this economy is a set of sequences for C_t, C_t', D_t, D_t', V_t, V_t', h_t, h_t', L_t, L_t', m_t, m_t', b_t, b_t', Lagrange multipliears and prices $P_{C,t}, P_{D,t}, w_t, q_t^h, q_t^d, S_t, R_t$ such that:

1. C_t, C_t', D_t, D_t', V_t, V_t', h_t, h_t', L_t, L_t', m_t, m_t', b_t, b_t' , as well as the corresponding Lagrange multipliers, solve the household problems for given prices and interest rate R_t.
2. $P_{C,t}, P_{D,t}, L_{C,t}, L_{D,t}$ solve the firms problem for given w_t
3. R_t is set according to (B.5.3), and S_t, R_t satisfy a no arbitrage condition.
4. Prices $P_{C,t}, P_{D,t}, w_t, q_t^h, q_t^d, S_t, R_t$ are such that all markets clear.

Parameterization

The exogenous credit limit model is parameterized using the values in Table B.2 below. Most of these are relatively standard and well within the range of estimates available in the literature. Two parameters, however, deserve further explanation. In the collateral constraint model of the next section, the value of the housing demand shifter, j, pins down the maximum loan-to-value ratio in the impatient household's budget constraint. To set the latter to the empirically plausible value of 75%, we need $j = 0.468$. For the sake of comparability across the two models, we then impose the same value for j here.

Our framework abstracts from investment, government spending and net exports. Accordingly, at the aggregate level, income is equal to expenditure and therefore we need to calibrate the debt to expenditure ratio $\bar{\Omega}/Y$. The household debt to disposable income ratio (or the private credit to GDP) in the two countries has averaged around 110% while household expenditure tends to represent about 60% of output, implying a debt to expenditure ratio of about 1.8. Finally, we assume there are equal shares of constrained and unconstrained households.

Parameter	Description	Value
$\theta/(1-\theta)$	elasticity of substitution between ND and D stock	4
σ	elasticity of intertemporal substitution	0.5
β^L, β^H	discount factor: mortgagors, outright owners	0.95, 0.99
$1/(\eta-1)$	Frisch elasticity of labor supply	2
μ	habits parameter	0.5
j	housing demand shifter	0.468
δ	depreciation rate durables	0.025
$\varepsilon_{C,D}$	elasticity of varieties	4
$\vartheta_{C,D}$	cost of adjusting prices	150
r_π, r_Y, r_R	Taylor rule: CPI, output, smoothing	1.5,.05,.6
ω_{IH}	share constrained households	50%
ω_{PH}	share unconstrained households	50%
$\bar{\Omega}/Y$	debt to expenditure ratio	1.8
$1/(1-\beta^H\rho)$	Benchmark long term debt duration	1 year

TABLE B.2: Calibration of the model.

B.5.5 The model with an endogenous collateral limit

We now assume that the housing stock h_t can be used as collateral, following Kiyotaki and Moore, 1997 and Iacoviello, (2005).[5] The credit constraint (B.8) now reads

$$S_t b_{t+1} \leq \phi E\left(q^h_{,t+1} h_t \frac{\pi_{t+1}}{R_t}\right) \tag{B.26}$$

where ϕ is the steady state loan-to-value (LTV) ratio, and $q^h_{,t+1}h_t$ is the real value of the housing stock at t+1.

There is a *rental market* through which households can rent (from/to others) housing services for one period at a rate p_t. Households are now also heterogeneous with respect to the utility they derive from renting.[6] Within the impatient households, there are now two groups of agents. We denote IHm those households who derive relatively higher utility from owned housing, and IHr those households who derive similar utility from renting or owning. The population shares of IHm, PH and IHr are *exogenous* and given by $(\omega_{IHm}, \omega_{PH}, 1-\omega_{IH}-\omega_{PH})$, consistent with the evidence in Section 4.2 that the shares of each housing tenure group do not vary with changes in monetary policy. But, importantly, the housing tenure choice (how much house to own vs. rent) will be endogenous.

[5]Note that we are not allowing the stock of durable goods V_t to be collateralizable. The reason for this is twofold. First, although there might be some durable goods in the data that are collateralizable, this is not typically the case, even for large durables such as vehicles. Second, we want to distinguish the role of durable goods from that of the housing stock.

[6]This way of modeling renting and owning is a simplification. One could think of this as a reduced form way of capturing life-cycle considerations which are not present in this class of model.

Housing utility (services) can be derived from housing owned or rented. Let $h_t \in \mathbb{R}^+$ be the housing stock *owned*, $s_t^- \in \mathbb{R}^+$ the housing rented *to* others, and $s_t^+ \in \mathbb{R}^+$ the housing stock rented *from* others. Housing services are then given by

$$\tilde{h}_{t,i} = h_{t,i} - s_{t,i}^- + \gamma_i s_{t,i}^+ \quad i \in \{PH, IHm, IHr\} \tag{B.27}$$

with $\gamma \in [0,1)$ capturing different reasons why households might preference to own rather than to rent.

Crucially, γ is household specific: $0 < \gamma_{IHm} = \gamma_{PH} < \gamma_{IHr} = 1$. This means that PH and IHm derive a higher marginal utility from their housing stock that is not rented out, $h_{t,i} - s_{t,i}^-$, than from the housing stock that they may rent from others, s_t^+. For IHr, on the other hand, the marginal utility is equal whether its owned or rented. All households face an individual housing *feasibility* constraint

$$h_{t,i} - s_{t,i}^- \geq 0 \tag{B.28}$$

meaning that they cannot rent out more than they currently own, $h_{t,i}$, and they cannot sub-let.[7]

Households also face a quadratic adjustment cost when adjusting the housing stock, given by

$$\xi_{h,t} = \phi_h \left(\frac{\Delta h_t}{h_{t-1}}\right)^2 \frac{q_t^h h_{t-1}}{2} \tag{B.29}$$

The impatient household (whether borrower or mortgagor/renter in equilibrium) solves the following optimization problem

$$W_{IH} = \max_{\left\{C_t, D_t, V_{t+1}, h_t, s_t^+, s_t^-, L_t, b_{t+1}, m_t\right\}} E_0 \sum_{t=0}^{\infty} \left(\beta^i\right)^t u\left(x_t, \tilde{h}_t, L_t\right)$$

[7] Note that restriction (B.28) together with the non-negativity of $s_{t,i}^-$ already imply that $h_{t,i} \geq 0$.

subject to (Lagrange multipliers in parenthesis)

$$
\begin{aligned}
C_t + q_t^d D_t + q_t^h \Delta h_t + \frac{b_t}{\pi_{c,t}} + \xi_{h,t} &= w_t L_t + S_t m_t + p_t\left(s_t^- - s_t^+\right) + T_t \quad (\lambda_t) \\
V_{t+1} &= \left(1 - \Phi\left(\frac{D_t}{D_{t-1}}\right)\right) D_t + (1-\delta) V_t \quad (\lambda_t q_t^v) \\
\tilde{h}_t &= \begin{cases} h_{t,i} + \gamma s_{t,i}^+ - s_{t,i}^- & \text{if } i = IHm \\ h_{t,i} + s_{t,i}^+ - s_{t,i}^- & \text{if } i = IHr \end{cases} \quad \left(\lambda_t^3\right) \\
h_t - s_t^- &\geq 0 \quad \left(\lambda_t^5\right) \\
s_t^- &\geq 0 \quad \left(\lambda_t^6\right) \\
s_t^+ &\geq 0 \quad \left(\lambda_t^7\right) \\
b_{t+1} &= m_t + \rho \frac{b_t}{\pi_t} \quad \left(\lambda_t^8\right) \\
S_t b_{t+1} &\leq \phi E\left(\frac{q_{,t+1}^h h_t \pi_{t+1}}{R_t}\right) \quad (\lambda_{BC,t})
\end{aligned}
$$

The patient household solves

$$
W_{PH} = \max_{\left\{C_t, D_t, V_{t+1}, h_t, s_t^+, s_t^-, L_t, b_{t+1}, m_t\right\}} E_0 \sum_{t=0}^{\infty} \left(\beta^H\right)^t u\left(x_t, \tilde{h}_t, L_t\right)
$$

subject to (Lagrange multipliers in parenthesis)

$$
\begin{aligned}
C_t + q_t^d D_t + q_t^h \Delta h_t - \frac{b_t}{\pi_{c,t}} + \xi_{h,t} &= w_t L_t - S_t m_t + p_t\left(s_t^- - s_t^+\right) + \Pi_{C,t} + \Pi_{D,c} + T_t \quad (\lambda_t) \\
V_{t+1} &= \left(1 - \Phi\left(\frac{D_t}{D_{t-1}}\right)\right) D_t + (1-\delta) V_t \quad (\lambda_t q_t^v) \\
\tilde{h}_t &= h_{t,i} + \gamma s_{t,i}^+ - s_{t,i}^- \quad \left(\lambda_t^3\right) \\
h_t - s_t^- &\geq 0 \quad \left(\lambda_t^5\right) \\
s_t^- &\geq 0 \quad \left(\lambda_t^6\right) \\
s_t^+ &\geq 0 \quad \left(\lambda_t^7\right) \\
b_{t+1} &= m_t + \rho \frac{b_t}{\pi_t} \quad \left(\lambda_t^8\right)
\end{aligned}
$$

Market clearing

Denote with $''$ (double tilde) the variables of IHr and with $'$ (single tilde) the ones for PH . Clearing of markets implies

$$\begin{aligned}
Y_{C,t} &= \omega_{IH}\left(C_t+\xi_{h,t}\right)+\omega_{PH}\left(C_t^{'}+\xi_{h,t}^{'}\right)+\left(1-\omega_{IH}-\omega_{PH}\right)C_t^{''}+\frac{\vartheta_C}{2}\left(\pi_{C,t}-1\right)^2 Y_{Ct} \\
Y_{D,t} &= \omega_{IH}D_t+\omega_{PH}D_t^{'}+\left(1-\omega_{IH}-\omega_{PH}\right)D_t^{''}+\frac{\vartheta_D}{2}\left(\pi_{D,t}-1\right)^2 Y_{Dt} \\
L_{C,t}+L_{D,t} &= \omega_{IH}L_t+\omega_{PH}L_t^{'}+\left(1-\omega_{IH}-\omega_{PH}\right)L_t^{''} \\
0 &= \omega_{IH}m_t+\omega_{PH}m_t^{'}+\left(1-\omega_{IH}-\omega_{PH}\right)m_t^{''} \\
H &= \omega_{IH}h_t+\omega_{PH}h_t^{'}+\left(1-\omega_{IH}-\omega_{PH}\right)h_t^{''} \\
s_t^{+}+s_t^{+'}+s_t^{+''} &= s_t^{-}+s_t^{-'}+s_t^{-''}
\end{aligned}$$

Housing tenure in steady state

In the next subsection B.5.9 we show that, for an owning-preference threshold $\bar{\gamma}$ such that $\gamma<\bar{\gamma}$, there exists zero-inflation steady state ($SS^{\pi=0}$) in which:[8]

1. The PHs own housing stock ($h^{'}>0$) and rent out part of it ($s^{-'}>0$).
2. Impatient renters do not own housing ($h^{''}=0$) which means they cannot: (i) borrow ($b^{''}=0$), or (ii) rent to others ($s^{-''}=0$). They instead rent housing services ($s^{+''}>0$).
3. The IHms own housing ($h>0$) but do not participate in the rental market ($s^{-}=s^{+}=0$).

Parameterization

In Table B.3, we report the values used for the collateral constraint model which are, in most cases, the same as in Table B.2. The steady state loan-to-value ratio, ϕ, (which did not appear in the previous parameterization) is set to 75%, consistent with the sample averages reported by Besley, Meads, and Surico, 2013. On the other hand, the shares of mortgagors and outright owners mimic the average values we observe in the FES/LCSF and CEX.

[8] We assume that the conditions above also hold for states "near the SS". This is equivalent to requiring that the wedge between the rental rate p_t and the house price q_t^h not to diverge "too much" from its value in the steady state $SS^{\pi=0}$. The assumption of no change in housing tenure following a monetary policy shock is, however, consistent with the results presented in Section 4.2.

Parameter	Description	Value
$\theta/(1-\theta)$	elasticity of substitution between ND and D stock	4
σ	elasticity of intertemporal substitution	0.5
β^L, β^H	discount factor: mortgagors, outright owners	0.95, 0.99
$1/(\eta-1)$	Frisch elasticity of labor supply	2
μ	habits parameter	0.5
j	housing demand shifter	0.468
δ	depreciation rate durables	0.025
$\varepsilon_{C,D}$	elasticity of varieties	4
$\vartheta_{C,D}$	cost of adjusting prices	150
r_π, r_Y, r_R	Taylor rule: CPI, output, smoothing	1.5,.05,.6
ω_{IH}	share of mortgagors	45%
ω_{PH}	share outright owners	35%
m	max LTV	0.75
$\bar{\Omega}/Y$	debt to expenditure ratio	1.8
$1/(1-\beta^H\rho)$	Benchmark long term debt duration	1 year

TABLE B.3: Calibration of the model.

Renters

Figure B.11 presents the impulse response functions for a 25bp cut in interest rates for non-durable consumption, durable expenditure and income of renters in the collateral constraint model. These responses are the analogue of the ones presented in Figure 2.10. Two interesting features are worth discussing. First, the renters' responses are relatively large, being only slightly smaller than those of mortgagors but larger than those of outright owners. The fact that renters' consumption responds significantly is intuitive given that these households are constrained and wealth-poor in our model.

Second, the renters' responses for non-durable consumption and for durable expenditure vary with the duration of mortgage contracts. While this may seem less intuitive, it can be understood by looking at the response of labor income. To the extent that different values for ρ produce different aggregate demand effects, these will also affect the renters' income.

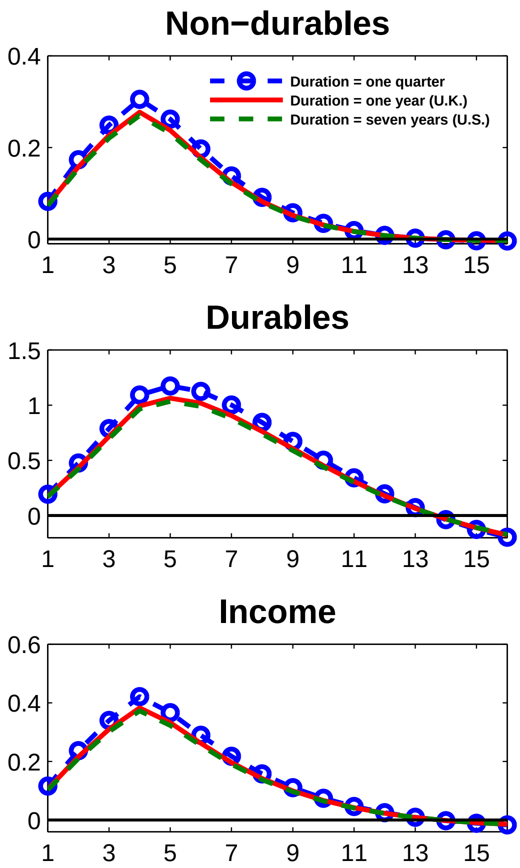


FIGURE B.11: Response of non-durable consumption, durable expenditure and income in a model with a collateral constraint: results for renters. Duration refers to the effective duration of fixed-rate mortgage contracts in the aggregate economy which, on average, is about one year for the U.K. and around 7 years for the U.S.

B.5.6 Log-Linearized Conditions: Exogenous Credit Limit

In this subsection, we present all the equilibrium conditions (budget constraints, optimality conditions and market clearing) log-linearized around a 0-inflation (in both sectors) SS. We want the AC for durables, $\Phi(\cdot)$, to have the following properties: $\Phi(SS) = \Phi'(SS) = 0$ and $\Phi''(SS) > 0$.

Aggregate Demand

$$
\begin{aligned}
\frac{Y_C}{Y}\hat{Y}_{C,t} &= \omega_{IH}\frac{C}{Y}\hat{C}_t + \omega_{PH}\frac{C'}{Y}\hat{C}'_t && \text{(B.30)}\\
\frac{Y_D}{Y}\hat{Y}_{D,t} &= \omega_{IH}\frac{D}{Y}\hat{D}_t + \omega_{PH}\frac{D'}{Y}\hat{D}'_t && \text{(B.31)}\\
\frac{L_C}{Y}\hat{L}_{C,t} + \frac{L_D}{Y}\hat{L}_{D,t} &= \omega_{IH}\frac{L}{Y}\hat{L}_t + \omega_{PH}\frac{L'}{Y}\hat{L}'_t && \text{(B.32)}\\
0 &= \hat{b}_t + \hat{b}'_t && \text{(B.33)}\\
\hat{\lambda}_t &= (1-\theta)\left(\hat{V}_t - \hat{C}_t\right) + \frac{\sigma}{1-\beta^L\mu}E_t\left(\mu\beta^L\hat{x}_{t+1} - \hat{x}_t\right) && \text{(B.34)}\\
\hat{\lambda}'_t &= (1-\theta)\left(\hat{V}'_t - \hat{C}'_t\right) + \frac{\sigma}{1-\beta^H\mu}E_t\left(\mu\beta^H\hat{x}'_{t+1} - \hat{x}'_t\right) && \text{(B.35)}\\
\hat{x}_t &= \frac{\theta}{1-\mu}\left(\hat{C}_t - \mu\hat{C}_{t-1}\right) + \frac{1-\theta}{1-\mu}\left(\hat{V}_t - \mu\hat{V}_{t-1}\right) && \text{(B.36)}\\
\hat{x}'_t &= \frac{\theta}{1-\mu}\left(\hat{C}'_t - \mu\hat{C}'_{t-1}\right) + \frac{1-\theta}{1-\mu}\left(\hat{V}'_t - \mu\hat{V}'_{t-1}\right) && \text{(B.37)}\\
\hat{V}_{t+1} &= \delta\hat{D}_t + (1-\delta)\hat{V}_t && \text{(B.38)}\\
\hat{V}'_{t+1} &= \delta\hat{D}'_t + (1-\delta)\hat{V}'_t && \text{(B.39)}\\
\hat{\lambda}_t + \hat{q}^v_t &= \left(1-\beta^L(1-\delta)\right)E_t\left(\hat{C}_{t+1} - \hat{V}_{t+1}\right) + (1-\delta)\beta^L E_t\left(\hat{q}^v_{t+1}\right) + E_t\left(\hat{\lambda}_{t+1}\right) && \text{(B.40)}\\
\hat{\lambda}'_t + \hat{q}^v_t &= \left(1-\beta^H(1-\delta)\right)E_t\left(\hat{C}'_{t+1} - \hat{V}'_{t+1}\right) + (1-\delta)\beta^H E_t\left(\hat{q}^v_{t+1}\right) + E_t\left(\hat{\lambda}'_{t+1}\right) && \text{(B.41)}\\
\frac{\hat{q}^d_t}{\Phi''} &= \frac{\hat{q}^v_t}{\Phi''} + \hat{D}_{t-1} + \beta^L E_t\left(\hat{D}_{t+1}\right) - (1+\beta^L)\hat{D}_t && \text{(B.42)}\\
\frac{\hat{q}^d_t}{\Phi''} &= \frac{\hat{q}^v_t}{\Phi''} + \hat{D}'_{t-1} + \beta^L E_t\left(\hat{D}'_{t+1}\right) - (1+\beta^L)\hat{D}'_t && \text{(B.43)}\\
\hat{\lambda}_t &= \frac{\beta^L}{\beta^H}E_t\left(\hat{\lambda}_{t+1} + \beta^L\rho\hat{S}_{t+1} - \hat{S}_t - \hat{\pi}_{C,t+1}\right) + \left(1 - \frac{\beta^L}{\beta^H}\right)\left(\hat{\lambda}_{CC,t} + \hat{R}_t\right) && \text{(B.44)}\\
\hat{\lambda}'_t &= E_t\left(\hat{\lambda}'_{t+1} + \beta'\rho\hat{S}_{t+1} - \hat{S}_t - \hat{\pi}_{C,t+1}\right) && \text{(B.45)}
\end{aligned}
$$

Housing and Renting Markets

$$
\begin{aligned}
\hat{\lambda}_t + \hat{q}^h_t + \phi_h\Delta\hat{h}_t &= \frac{j}{\lambda q^h h}\left(\hat{j}_t - \hat{h}_t\right) + E_t\Bigg(\beta^L\left(\hat{\lambda}_{t+1} + \hat{q}^h_{t+1} + \phi_h\Delta\hat{h}_{t+1}\right) + && \text{(B.46)}\\
\hat{\lambda}'_t + \hat{q}^h_t + \phi_h\Delta\hat{h}'_t &= \frac{j}{\lambda q^h h'}\left(\hat{j}_t - \hat{h}'_t\right) + E_t\Bigg(\beta^H\left(\hat{\lambda}'_{t+1} + \hat{q}^h_{t+1} + \phi_h\Delta\hat{h}'_{t+1}\right)\Bigg) && \text{(B.47)}\\
0 &= \omega_{IH}\frac{q^h h}{Y}\hat{h}_t + \omega_{PH}\frac{q^h h'}{Y}\hat{h}'_t && \text{(B.48)}
\end{aligned}
$$

Borrowing constraint and Evolution of Debt

$$\hat{S}_t + \hat{b}_t = -\left(\hat{R}_t - E_t\left(\hat{\pi}_{c,t+1}\right)\right) \tag{B.49}$$

$$\hat{b}_t = (1-\rho)\hat{m}_t + \rho\left(\hat{b}_{t-1} - \hat{\pi}_{c,t}\right) \tag{B.50}$$

Aggregate Supply

$$\hat{Y}_{C,t} = \hat{L}_{C,t} \tag{B.51}$$

$$\hat{Y}_{D,t} = \hat{L}_{D,t} \tag{B.52}$$

$$\frac{\Pi_C}{Y}\hat{\Pi}_{C,t} = \frac{L_C}{Y}\hat{L}_{C,t} - \frac{L_C}{Y}\left(\hat{w}_t + \hat{L}_{C,t}\right) \tag{B.53}$$

$$\frac{\Pi_D}{Y}\hat{\Pi}_{D,t} = q^d\frac{L_D}{Y}\left(\hat{q}_t^d + \hat{L}_{D,t}\right) - w\frac{L_D}{Y}\left(\hat{w}_t + \hat{L}_{D,t}\right) \tag{B.54}$$

$$(\eta - 1)\hat{L}_t = \hat{\lambda}_t + \hat{w}_t \tag{B.55}$$

$$(\eta' - 1)\hat{L}_t' = \hat{\lambda}_t' + \hat{w}_t \tag{B.56}$$

$$\hat{\pi}_{C,t} = \beta' E_t\left(\hat{\pi}_{C,t+1}\right) + \frac{\varepsilon_C - 1}{\vartheta_C}\hat{mc}_{C,t} \tag{B.57}$$

$$\hat{\pi}_{D,t} = \beta' E_t\left(\hat{\pi}_{D,t+1}\right) + \frac{\varepsilon_D - 1}{\vartheta_D}\hat{mc}_{D,t} \tag{B.58}$$

$$m\hat{c}_{C,t} = \hat{w}_t \tag{B.59}$$

$$\hat{mc}_{D,t} = \frac{\varepsilon_D - 1}{\varepsilon_D}\left(\hat{w}_t - \hat{q}_t^d\right) \tag{B.60}$$

Evolution of Variables and Budget Constraints

$$\hat{V}_{t+1} = \delta\hat{D}_t + (1-\delta)\hat{V}_t \tag{B.61}$$

$$\hat{V}_{t+1}' = \delta\hat{D}_t' + (1-\delta)\hat{V}_t' \tag{B.62}$$

$$\begin{aligned}\frac{S(1-\rho)b}{Y}\left(\hat{S}_t + \hat{m}_t\right) &= \frac{C}{Y}\hat{C}_t + \frac{q^d D}{Y}\left(\hat{D}_t + \hat{q}_t^d\right) + \frac{q^h h}{Y}\Delta\hat{h}_t + \frac{b}{Y}\left(\hat{b}_{t-1} - \hat{\pi}_{C,t}\right) - \\ &\quad - \frac{wL}{Y}\left(\hat{w}_t + \hat{L}_t\right)\end{aligned} \tag{B.63}$$

$$\begin{aligned}\frac{b'}{Y}\left(\hat{b}_{t-1}' - \hat{\pi}_{C,t}\right) &= \frac{C'}{Y}\hat{C}_t' + \frac{q^d D'}{Y}\left(\hat{D}_t' + \hat{q}_t^d\right) + \frac{q^h h'}{Y}\Delta\hat{h}_t' + \frac{S(1-\rho)b'}{Y}\left(\hat{S}_t + \hat{m}_t'\right) - \\ &\quad - \frac{wL'}{Y}\left(\hat{w}_t + \hat{L}_t'\right) - \frac{\Pi_C}{Y}\hat{\Pi}_{C,t} - \frac{\Pi_D}{Y}\hat{\Pi}_{D,t}\end{aligned} \tag{B.64}$$

Monetary Policy

$$\hat{R}_t = (1 - r_R)(1 + r_\pi)\hat{\pi}_{t-1} + r_Y(1 - r_R)\hat{Y}_{t-1} + r_R\hat{R}_{t-1} + \hat{\epsilon}_{R,t} + \rho_\epsilon\hat{\epsilon}_{R,t-1} \tag{B.65}$$

$$\hat{Y}_t = \alpha_Y\hat{Y}_{C,t} + (1 - \alpha_Y)\hat{Y}_{D,t} \tag{B.66}$$

$$\hat{\pi}_t = \alpha_\pi\hat{\pi}_{C,t} + (1 - \alpha_\pi)\hat{\pi}_{D,t} \tag{B.67}$$

B.5.7 Log-Linearized Conditions: Collateral Constraint Model

In what follows, we linearize the main equilibrium conditions around a 0-inflation (in both sectors) steady state ($SS^{\pi=0}$) in which:

1. PH own housing stock ($h^{'} > 0$) and rents out part of it ($s^{-'} > 0$)
2. Impatient renters do not own housing ($h^{''} = 0$) which means they cannot: (i) borrow ($b^{''} = 0$), and (ii) rent to others ($s^{-''} = 0$). They instead rent housing services from others ($s^{+''} > 0$)
3. IHm own housing ($h > 0$) but do not participate in the renting market ($s^{-} = s^{+} = 0$)

In such a SS, therefore, the following must hold:

1. $\lambda^{5'} = \lambda^{6'} = 0$; $\lambda^{7'} \geq 0$
2. $\lambda^{5''} \geq 0$; $\lambda^{6''} \geq 0$; $\lambda^{7''} = 0$
3. $\lambda^{5} = 0$; $\lambda^{6} \geq 0$; $\lambda^{7} \geq 0$

In the next section, we show that such SS exists for an owning-preference $\gamma < \bar{\gamma}$. We assume that the conditions above also hold for states **"near the SS"**. This is not a trivial assumption. In escence, this assumption requires the wedge between the rental rate p_t and the house price q_t^h not to diverge "too much" from its value in the steady state $SS^{\pi=0}$ described above. As shown below, the wedge in such steady state,

$$p = \left(1 - \beta^{'}\right) q^h$$

Aggregate Demand

$$\frac{Y_C}{Y}\hat{Y}_{C,t} = \omega_{IH}\frac{C}{Y}\hat{C}_t + \omega_{PH}\frac{C'}{Y}\hat{C}_t' + (1-\omega_{IH}-\omega_{PH})\frac{C''}{Y}\hat{C}_t'' \tag{B.68}$$

$$\frac{Y_D}{Y}\hat{Y}_{D,t} = \omega_{IH}\frac{D}{Y}\hat{D}_t + \omega_{PH}\frac{D'}{Y}\hat{D}_t' + (1-\omega_{IH}-\omega_{PH})\frac{D''}{Y}\hat{D}_t'' \tag{B.69}$$

$$\frac{L_C}{Y}\hat{L}_{C,t} + \frac{L_D}{Y}\hat{L}_{D,t} = \omega_{IH}\frac{L}{Y}\hat{L}_t + \omega_{PH}\frac{L'}{Y}\hat{L}_t' + (1-\omega_{IH}-\omega_{PH})\frac{L''}{Y}\hat{L}_t'' \tag{B.70}$$

$$0 = \hat{b}_t + \hat{b}_t' \tag{B.71}$$

$$\hat{\lambda}_t = (1-\theta)\left(\hat{V}_t - \hat{C}_t\right) + \frac{\sigma}{1-\beta^L\mu}E_t\left(\mu\beta^L\hat{x}_{t+1} - \hat{x}_t\right) \tag{B.72}$$

$$\hat{\lambda}_t' = (1-\theta)\left(\hat{V}_t' - \hat{C}_t'\right) + \frac{\sigma}{1-\beta^H\mu}E_t\left(\mu\beta^H\hat{x}_{t+1}' - \hat{x}_t'\right) \tag{B.73}$$

$$\hat{\lambda}_t'' = (1-\theta)\left(\hat{V}_t'' - \hat{C}_t''\right) + \frac{\sigma}{1-\beta^L\mu}E_t\left(\mu\beta^L\hat{x}_{t+1}'' - \hat{x}_t''\right) \tag{B.74}$$

$$\hat{x}_t = \frac{\theta}{1-\mu}\left(\hat{C}_t - \mu\hat{C}_{t-1}\right) + \frac{1-\theta}{1-\mu}\left(\hat{V}_t - \mu\hat{V}_{t-1}\right) \tag{B.75}$$

$$\hat{x}_t' = \frac{\theta}{1-\mu}\left(\hat{C}_t' - \mu\hat{C}_{t-1}'\right) + \frac{1-\theta}{1-\mu}\left(\hat{V}_t' - \mu\hat{V}_{t-1}'\right) \tag{B.76}$$

$$\hat{x}_t'' = \frac{\theta}{1-\mu}\left(\hat{C}_t'' - \mu\hat{C}_{t-1}''\right) + \frac{1-\theta}{1-\mu}\left(\hat{V}_t'' - \mu\hat{V}_{t-1}''\right) \tag{B.77}$$

$$\hat{V}_{t+1} = \delta\hat{D}_t + (1-\delta)\hat{V}_t \tag{B.78}$$

$$\hat{V}_{t+1}' = \delta\hat{D}_t' + (1-\delta)\hat{V}_t' \tag{B.79}$$

$$\hat{V}_{t+1}'' = \delta\hat{D}_t'' + (1-\delta)\hat{V}_t'' \tag{B.80}$$

$$\hat{\lambda}_t + \hat{q}_t^v = \left(1-\beta^L(1-\delta)\right)E_t\left(\hat{C}_{t+1} - \hat{V}_{t+1}\right) + (1-\delta)\beta^L E_t\left(\hat{q}_{t+1}^v\right) + E_t\left(\hat{\lambda}_{t+1}\right) \tag{B.81}$$

$$\hat{\lambda}_t' + \hat{q}_t^v = \left(1-\beta^H(1-\delta)\right)E_t\left(\hat{C}_{t+1}' - \hat{V}_{t+1}'\right) + (1-\delta)\beta^H E_t\left(\hat{q}_{t+1}^v\right) + E_t\left(\hat{\lambda}_{t+1}'\right) \tag{B.82}$$

$$\hat{\lambda}_t'' + \hat{q}_t^v = \left(1-\beta^L(1-\delta)\right)E_t\left(\hat{C}_{t+1}'' - \hat{V}_{t+1}''\right) + (1-\delta)\beta^L E_t\left(\hat{q}_{t+1}^v\right) + E_t\left(\hat{\lambda}_{t+1}''\right) \tag{B.83}$$

$$\frac{\hat{q}_t^d}{\Phi''} = \frac{\hat{q}_t^v}{\Phi''} + \hat{D}_{t-1} + \beta^L E_t\left(\hat{D}_{t+1}\right) - (1+\beta^L)\hat{D}_t \tag{B.84}$$

$$\frac{\hat{q}_t^d}{\Phi''} = \frac{\hat{q}_t^v}{\Phi''} + \hat{D}_{t-1}' + \beta^H E_t\left(\hat{D}_{t+1}'\right) - (1+\beta^H)\hat{D}_t' \tag{B.85}$$

$$\frac{\hat{q}_t^d}{\Phi''} = \frac{\hat{q}_t^v}{\Phi''} + \hat{D}_{t-1}'' + \beta^L E_t\left(\hat{D}_{t+1}''\right) - (1+\beta^L)\hat{D}_t'' \tag{B.86}$$

$$\hat{\lambda}_t = \frac{\beta^L}{\beta^H}E_t\left(\hat{\lambda}_{t+1} + \beta^H\rho\hat{S}_{t+1} - \hat{S}_t - \hat{\pi}_{C,t+1}\right) + \left(1-\frac{\beta^L}{\beta^H}\right)\left(\hat{\lambda}_{CC,t} + \hat{R}_t\right) \tag{B.87}$$

$$\hat{\lambda}_t' = E_t\left(\hat{\lambda}_{t+1}' + \beta^H\rho\hat{S}_{t+1} - \hat{S}_t - \hat{\pi}_{C,t+1}\right) \tag{B.88}$$

Housing and Renting Markets

$$\hat{\lambda}_t + \hat{q}_t^h + \phi_h \Delta \hat{h}_t = \frac{j}{\lambda q^h h}\left(\hat{j}_t - \hat{h}_t\right) + E_t\Bigg(\beta^L\left(\hat{\lambda}_{t+1} + \hat{q}_{t+1}^h + \phi_h \Delta \hat{h}_{t+1}\right) + \tag{B.89}$$

$$+\left(\beta^H - \beta^L\right)\phi\left(\hat{\lambda}_{CC,t} + \hat{q}_{t+1}^h + \hat{\pi}_{C,t+1}\right)\Bigg)$$

$$\hat{\lambda}_t'' + \hat{q}_t^h = \frac{j}{\lambda'' q^h s^{+''}}\left(\hat{j}_t - \hat{s}_t^{+''}\right) + E_t\Bigg(\beta^L\left(\hat{\lambda}_{t+1}'' + \hat{q}_{t+1}^h\right) + \tag{B.90}$$

$$+\left(\beta^H - \beta^L\right)\phi\left(\hat{\lambda}_{CC,t}'' + \hat{q}_{t+1}^h + \hat{\pi}_{C,t+1}\right)\Bigg) +$$

$$+\left(\left(1 - \beta^L - (\beta^H - \beta^L)\phi\right) - \frac{j}{\lambda'' q^h s^{+''}}\right)\tilde{\lambda}^{5''}$$

$$\hat{\lambda}_t' + \hat{q}_t^h + \phi_h \Delta \hat{h}_t' = \frac{j}{\lambda q^h (h' - s^{-'})}\left(\hat{j}_t - \hat{h}_t' + \hat{s}_t^{-'}\right) + E_t\left(\beta^H\left(\hat{\lambda}_{t+1}' + \hat{q}_{t+1}^h + \phi_h \Delta \hat{h}_{t+1}'\right)\right) \tag{B.91}$$

$$\hat{p}_t + \hat{\lambda}_t = \frac{j\gamma}{(1-\beta^H)\lambda q^h h}\left(\hat{j}_t - \hat{h}_t\right) + \left(1 - \frac{j\gamma}{(1-\beta^H)\lambda q^h h}\right)\hat{\lambda}_t^7 \tag{B.92}$$

$$\hat{p}_t + \hat{\lambda}_t = \frac{j}{(1-\beta^H)\lambda q^h h}\left(\hat{j}_t - \hat{h}_t\right) - \left(1 - \frac{j}{(1-\beta^H)\lambda q^h h}\right)\hat{\lambda}_t^6 \tag{B.93}$$

$$\hat{p}_t + \hat{\lambda}_t'' = \frac{j}{(1-\beta^H)\lambda'' q^h s^{+''}}\left(\hat{j}_t - \hat{s}_t^{+''}\right) \tag{B.94}$$

$$\hat{p}_t + \hat{\lambda}_t' = \frac{j\gamma}{(1-\beta^H)\lambda' q^h (h' - s^{-'})}\left(\hat{j}_t - \hat{h}_t' + \hat{s}_t^{-'}\right) + \tag{B.95}$$

$$+\left(1 - \frac{j\gamma}{(1-\beta^H)\lambda' q^h (h' - s^{-'})}\right)\hat{\lambda}_t^{7'}$$

$$\hat{p}_t + \hat{\lambda}_t' = \frac{j}{(1-\beta^H)\lambda' q^h (h' - s^{-'})}\left(\hat{j}_t - \hat{h}_t' + \hat{s}_t^{-'}\right) \tag{B.96}$$

$$0 = \omega_{IH}\frac{q^h h}{Y}\hat{h}_t + \omega_{PH}\frac{q^h h'}{Y}\hat{h}_t' \tag{B.97}$$

$$\hat{s}_t^{+''} = \hat{s}_t^{-'} \tag{B.98}$$

Borrowing constraint and Evolution of Debt

$$\hat{S}_t + \hat{b}_{t+1} = E_t\left(\hat{q}_{t+1}^h\right) + \hat{h}_t - \hat{rr}_t \tag{B.99}$$

$$\hat{rr}_t = \hat{R}_t - E_t\left(\hat{\pi}_{C,t+1}\right) \tag{B.100}$$

$$\hat{b}_{t+1} = (1-\rho)\hat{m}_t + \rho\left(\hat{b}_t - \hat{\pi}_{c,t}\right) \tag{B.101}$$

Aggregate Supply

$$\hat{Y}_{C,t} = \hat{L}_{C,t} \tag{B.102}$$
$$\hat{Y}_{D,t} = \hat{L}_{D,t} \tag{B.103}$$
$$\frac{\Pi_C}{Y}\hat{\Pi}_{C,t} = \frac{L_C}{Y}\hat{L}_{C,t} - \frac{L_C}{Y}\left(\hat{w}_t + \hat{L}_{C,t}\right) \tag{B.104}$$
$$\frac{\Pi_D}{Y}\hat{\Pi}_{D,t} = q^d\frac{L_D}{Y}\left(\hat{q}_t^d + \hat{L}_{D,t}\right) - w\frac{L_D}{Y}\left(\hat{w}_t + \hat{L}_{D,t}\right) \tag{B.105}$$
$$(\eta - 1)\hat{L}_t = \hat{\lambda}_t + \hat{w}_t \tag{B.106}$$
$$(\eta^{'} - 1)\hat{L}_t^{'} = \hat{\lambda}_t^{'} + \hat{w}_t \tag{B.107}$$
$$(\eta^{''} - 1)\hat{L}_t^{''} = \hat{\lambda}_t^{''} + \hat{w}_t \tag{B.108}$$
$$\hat{\pi}_{C,t} = \beta^H E_t\left(\hat{\pi}_{C,t+1}\right) + \frac{\varepsilon_C - 1}{\vartheta_C}\hat{mc}_{C,t} \tag{B.109}$$
$$\hat{\pi}_{D,t} = \beta^H E_t\left(\hat{\pi}_{D,t+1}\right) + \frac{\varepsilon_D - 1}{\vartheta_D}\hat{mc}_{D,t} \tag{B.110}$$
$$m\hat{c}_{C,t} = \hat{w}_t \tag{B.111}$$
$$\hat{mc}_{D,t} = \frac{\varepsilon_D - 1}{\varepsilon_D}\left(\hat{w}_t - \hat{q}_t^d\right) \tag{B.112}$$

Evolution of Variables and Budget Constraints

$$\hat{V}_{t+1} = \delta\hat{D}_t + (1-\delta)\hat{V}_t \tag{B.113}$$
$$\hat{V}_{t+1}^{'} = \delta\hat{D}_t^{'} + (1-\delta)\hat{V}_t^{'} \tag{B.114}$$
$$\hat{V}_{t+1}^{''} = \delta\hat{D}_t^{''} + (1-\delta)\hat{V}_t^{''} \tag{B.115}$$
$$\frac{S(1-\rho)b}{Y}\left(\hat{S}_t + \hat{l}_t\right) = \frac{C}{Y}\hat{C}_t + \frac{q^d D}{Y}\left(\hat{D}_t + \hat{q}_t^d\right) + \frac{q^h h}{Y}\Delta\hat{h}_t + \frac{b}{Y}\left(\hat{b}_{t-1} - \hat{\pi}_{C,t}\right) - \frac{wL}{Y}\left(\hat{w}_t + \hat{L}_t\right) \tag{B.116}$$
$$\begin{aligned}\frac{b^{'}}{Y}\left(\hat{b}_{t-1}^{'} - \hat{\pi}_{C,t}\right) &= \frac{C^{'}}{Y}\hat{C}_t^{'} + \frac{q^d D^{'}}{Y}\left(\hat{D}_t^{'} + \hat{q}_t^d\right) + \frac{q^h h^{'}}{Y}\Delta\hat{h}_t^{'} + \frac{S(1-\rho)b^{'}}{Y}\left(\hat{S}_t + \hat{l}_t^{'}\right) - \\ &\quad -\frac{wL^{'}}{Y}\left(\hat{w}_t + \hat{L}_t^{'}\right) - \frac{(1-\beta^{'})q^h s^{-'}}{Y}\left(\hat{p}_t + \hat{s}_t^{-'}\right) - \frac{\Pi_C}{Y}\hat{\Pi}_{C,t} - \frac{\Pi_D}{Y}\hat{\Pi}_{D,t}\end{aligned} \tag{B.117}$$
$$0 = \frac{C^{''}}{Y}\hat{C}^{''} + \frac{q^d D^{''}}{Y}\left(\hat{D}_t^{''} + \hat{q}_t^d\right) + \frac{(1-\beta^{'})q^h s^{+''}}{Y}\left(\hat{p}_t + \hat{s}_t^{+''}\right) - \frac{wL^{''}}{Y}\left(\hat{w}_t + \hat{L}_t^{''}\right) \tag{B.118}$$

Monetary Policy

$$\hat{R}_t = (1 - r_R)\left(1 + r_\pi\right)\hat{\pi}_{t-1} + r_Y\left(1 - r_R\right)\hat{Y}_{t-1} + r_R\hat{R}_{t-1} + \hat{\epsilon}_{R,t} + \rho_\epsilon\hat{\epsilon}_{R,t-1} \tag{B.119}$$
$$\hat{Y}_t = \alpha_Y\hat{Y}_{C,t} + \left(1 - \alpha_Y\right)\hat{Y}_{D,t} \tag{B.120}$$
$$\hat{\pi}_t = \alpha_\pi\hat{\pi}_{C,t} + \left(1 - \alpha_\pi\right)\hat{\pi}_{D,t} \tag{B.121}$$

B.5.8 Relevant Steady State Ratios

Assuming zero inflation in both sectors, we can characterize some of the steady state (SS) ratios, quantities and relative prices.

Relative (household) consumption $\frac{C}{D}$

From the first order condition for durable stock of household $i \in \{IH,\, PH,\, Renter\}$ we have that

$$\lambda^i q^v = \beta^i \lambda^i \left(\frac{1-\theta}{\theta} \frac{C}{V} + (1-\delta) q^v \right) \tag{B.122}$$

In addition, we can compute the durable investment in steady state from the law of movement for the durable stock:

$$V = \frac{D}{\delta} \Leftrightarrow V\delta = D$$

Finally, using the FOC for durable investemtn in steady state, we have

$$q^v = q^d = \frac{\varepsilon_D - 1}{\varepsilon_D} \frac{\varepsilon_C}{\varepsilon_C - 1}$$

where the second equality holds in a zero inflation, flexible-prices SS. We can then write the retlative consumption of durables as

$$\frac{C}{D} = \frac{\theta}{1-\theta} \frac{\varepsilon_C}{\varepsilon_D} \frac{(\varepsilon_D - 1)}{(\varepsilon_C - 1)} \frac{(1 - \beta^i(1-\delta))}{\beta^i \delta} \tag{B.123}$$

Note that (B.123) is independent of credit frictions. This is true because we are assuming that durables *cannot* be used as collateral for borrowing. Given that preferences are homothetic, $\frac{C}{D}$ is also independent of income.

Hours worked L

From the labor-consumption decision in SS

$$\lambda^i \frac{\varepsilon_C - 1}{\varepsilon_C} = L^{\eta^i - 1} \tag{B.124}$$

while from the optimality condition for non durable consumption we have that

$$\lambda^i = \theta(1 - \beta^i \mu) \left(\frac{C}{D} \delta \right)^{\theta - 1} x^{-\sigma} \tag{B.125}$$

Combining (B.123), (B.124) and (B.125), and for given values of $\varepsilon_C, \varepsilon_D, \theta, \mu, \delta, \beta^i, \sigma$ we can back up the SS hours worked as a function of the elasticity η^i and the consumption basket x:

$$L = \left(\frac{\varepsilon_C - 1}{\varepsilon_C} \theta (1 - \beta^i \mu) \left(\frac{C}{D} \delta \right)^{\theta - 1} x^{-\sigma} \right)^{\frac{1}{\eta^i - 1}} \tag{B.126}$$

Housing value: $q^v h$

For the impatient household (IH), from the housing-consumption optimality condition and given that $\frac{\partial \xi_h}{\partial h}|_{SS} = 0$, we have

$$\lambda q^h = \frac{j}{h} + \beta \left(\lambda q^h + m \lambda q^h (\beta' - \beta) \right)$$

which implies

$$q^h h = \frac{j}{\lambda \left(1 - \beta(1 + m(\beta' - \beta))\right)} \tag{B.127}$$

For patient hpuseholds (PH) we have

$$\lambda q^h = \frac{j}{h'} + \beta' \left(\lambda' q^h \right)$$

and therefore

$$q^h h' = \frac{j}{\lambda' \left(1 - \beta'\right)} \tag{B.128}$$

Debt: b

Given that the borrowing constraint binds in a steady state equilibrium,we have

$$\begin{aligned} b &= m\beta' q^h h \\ &= m\beta' \frac{j}{\lambda \left(1 - \beta(1 + m(\beta' - \beta))\right)} \end{aligned} \tag{B.129}$$

Durable Investment over Output: $\frac{D}{Y}$

From the durable intertemporal condition of each household $i \in$ {IH, PH, Renters} , and using the fact that in SS $q^d = q^v$, we have

$$\lambda^i q^d = \beta^i \left(\lambda^i \frac{1-\theta}{\theta} \frac{C^i}{D^i} \delta + (1-\delta) q^d \right)$$

from where we can derive

$$\frac{q^d D^i}{Y} = \frac{C^i}{Y} \left(\frac{\beta \delta (1-\theta) \lambda^i}{\theta \left(\lambda^i - \beta(1-\delta) \right)} \right) \tag{B.130}$$

Consumption to Output share: $\frac{C}{Y}$

For the impatient household, the budget in SS reads

$$\frac{C}{Y} + \frac{q^d D}{Y} + \left(\frac{1-\beta'}{\beta'}\right)\frac{b}{Y} = \frac{wL}{Y} \tag{B.131}$$

and, similarly, for the renters we have

$$\frac{C''}{Y} + \frac{q^d D''}{Y} = \frac{wL''}{Y} \tag{B.132}$$

from which we can solve for $\frac{C}{Y}\frac{C''}{Y}$.

For the patient hoseholds we have

$$\frac{C}{Y} + \frac{q^d D}{Y} + \left(\frac{1-\beta'}{\beta'}\right)\frac{b}{Y} = \frac{wL}{Y} + \frac{\Pi_C}{Y} + \frac{\Pi_D}{Y} \tag{B.133}$$

whcih means we also need to solve fo $\frac{\Pi_C}{Y}$ and $\frac{\Pi_D}{Y}$. In SS,

$$\begin{aligned} \frac{\Pi_C}{Y} &= \frac{L_C}{Y}(1-w) && \text{(B.134)} \\ \frac{\Pi_D}{Y} &= \frac{L_D}{Y}\left(q^d - w\right) && \text{(B.135)} \\ &= \left(1 - \frac{L_C}{Y}\right)\left(q^d - w\right) \end{aligned}$$

Since we know how to solve for $\frac{C}{Y}$ and $\frac{C''}{Y}$, we can write

$$\frac{L_C}{Y} = \omega_{IH}\frac{C}{Y} + \omega_{PH}\frac{C'}{Y} + (1-\omega_{IH}-\omega_{PH})\frac{C''}{Y} \tag{B.136}$$

which means that both (B.134) and (B.135) can be written as a function of $\frac{C'}{Y}$ and deep parameters. From the budget constraint (B.133) we can then solve for $\frac{C'}{Y}$.

Output Shares: $\frac{Y_C}{Y}$ **and** $\frac{Y_D}{Y}$

Once we have solved for $\frac{C}{Y}$, $\frac{C'}{Y}$ and $\frac{C'}{Y}$ we can use eq. (B.136) to find $\frac{L_C}{Y}$ and

$$\frac{L_D}{Y} = 1 - \frac{L_C}{Y}$$

B.5.9 Proof of "Separation" In Steady State

Here we show that there is a steady state (SS) in which:

1. PH own housing stock ($h' > 0$) and rents out part of it ($s^{-'} > 0$)

2. Impatient renters do not own housing ($h^{''} = 0$) which means they cannot: (i) borrow ($b^{''} = 0$), and (ii) rent to others ($s^{-''} = 0$). They instead rent housing services from others ($s^{+''} > 0$)

3. IHm own housing ($h > 0$) but do not participate in the renting market ($s^{-} = s^{+} = 0$)

Intuition of the Proof:

At the SS interest rate $R = \frac{1}{\beta'}$, both IHm and IHr households want to borrow in order to shift consumption from the future to the current period. To do so, due to frictions in credit markets, they need to own housing stock to use as collateral. A crucial element is the LTV being <1, or the "haircut" on the value of the collateral being >0. This will imply that an increase in the amount of collateralizable asset (in this case housing stock) they own, translates into a less than proportional increase in the resources available to borrow. The SS relative rental rate $\frac{p}{q^h} = 1 - \beta^{'}$ (determined such that assets have the same returns) implies that, given the perfect substituability between renting and owning in terms of utility, a positive downpayment (m<1) will make it suboptimal for the IHr to own (i.e. invest) in *any* amount housing stock. For the IHm, however, a sufficiently high "bias for owning", reflected in a sufficiently low γ, implies they can overcome the relatively high $\frac{q^h}{p}$ by buying a house "smaller" than the one they would otherwise rent, and using it to get a collateralized loan (a mortgage).

What the above means is that the collateral value of a house *per se* is not enough incentive for the impatient households to invest / buy a house. This is true more in general in this kind of set-ups where the assets are priced by the patient (unconstrained) guys, including Iacoviello (2005). What we are adding in our set-up is the assumption that, for some households, the services provided by a rented house are less valuable in terms of utility than the services provided by an own house.

The arguments above make use of the fact that owning a house and renting are substitutes in terms of the utility generated. This implies that an increase of Δ units of housing stock h generates the *same* increase in utility as an increase of $\frac{1}{\gamma}\Delta$ units of rented housing s^{+} (with $\gamma = 1$ in the case of IHr households):

$$u\left(h + \Delta + \gamma s^{+} - s^{-}\right) = u\left(h + \gamma\left(s^{+} + \frac{1}{\gamma}\Delta\right) - s^{-}\right) \tag{B.137}$$

The idea of the proof is to show that, in a SS equilibrium with active mortgage markets, the relative rental rate needs to be

$$p = q^h\left(1 - \beta^{'}\right)$$

in which case the PH will be indifferent between lending and buy-to-rent, and under which IHm will want to own for a sufficiently low γ. For $p > q^h\left(1 - \beta^{'}\right)$, the PH would prefere not to lend funds through the mortgage martket, but to buy-to-let as much as possible. For such relative prices, we will see that the return of a buy-to-let strategy is such that $R^{buy-to-let} > R$. This would imply a collapse of the mortgage market and unbounded consumption growth, unless it bring the housing price up and back to $p = q^h\left(1-\beta^{'}\right)$. For $p < q^h\left(1-\beta^{'}\right)$, it will be the case that neither PH nor IHm will be willing to engage in buy-to-let, and therefore no household will be willing to rent to IHr.

Patient Households' Problem

Recall the relevant optimality conditions of the PH for the housing and renting variables, asuming for the time being that there are no adjustment cost to changing housin

$$\begin{aligned}
\lambda_t^{'} q_t^h &= \frac{j_t}{\tilde{h}_t^{'}} + \beta^{'} E_t\left(\lambda_{t+1}^{'} q_{t+1}^h\right) + \lambda_t^{5'} \\
p_t \lambda_t^{'} &= \frac{j_t \gamma}{\tilde{h}_t^{'}} + \lambda_t^{7'} \\
p_t \lambda_t^{'} &= \frac{j_t}{\tilde{h}_t^{'}} + \lambda_t^{5'} - \lambda_t^{6'}
\end{aligned}$$

In SS these conditions imply

$$\lambda^{'} q\left(1-\beta^{'}\right) = \frac{j}{\tilde{h}^{'}} + \lambda^{5'} \tag{B.138}$$

$$\lambda^{'} p = \frac{j\gamma}{\tilde{h}^{'}} + \lambda^{7'} \tag{B.139}$$

$$\lambda^{'} p = \frac{j}{\tilde{h}^{'}} + \lambda^{5'} - \lambda^{6'} \tag{B.140}$$

I will now consider different cases regarding the choice of (h, s^+, s^-) in SS, and analyze their pricing and market clearing implications.

- Case I: Assume $\boldsymbol{s^{+'} = 0}$

  This implies that, due to Inada conditions, $h^{'} - s^{-'} > 0$ and therefore $\lambda^{5'} = 0, \lambda^{7'} \geq 0$ from the KT conditions. Now, we have two possibilities:

  - $\boldsymbol{\lambda^{6'} = 0}$: From equations (B.138) and (B.140) we then have that

    $$p = q^h\left(1-\beta^{'}\right) \tag{B.141}$$

    while the KT conditions require $s^{-'} \geq 0$. Is the relative rental rate (B.141) consistent with $s^{+'} = 0$ as assume above? To check this, assume that, while in SS, the household switches, once and for all, from 1 unit of rented housing to γ units of owned housing that are not rented out to other households, so that housing utility $u\left(\tilde{h}^{'}\right)$ remains the same. The resource cost at t is simply

γq^h while the resource benefit at t and in the future are the rents not payed, $q^h\left(1-\beta'\right)$. The net present value (NPV), or net resource gain is

$$\begin{aligned} NPV &= q^h\left(1-\beta'\right)-\gamma q^h+\beta'\sum_{\tau=0}^{\infty}\beta' q^h\left(1-\beta'\right) \\ &= q^h\left(1-\gamma\right) \end{aligned}$$

which is >0 for any value of γ.
What are the choices of IHr and IHm households given the relative prices in (B.141)? Consider the case of the IHm (the case of IHr is similar but with $\gamma=1$). As above, sssume that the household switches, once and for all, from 1 unit of rented housing to γ units of owned housing that are not rented out to other households, so that housing utility $u\left(\tilde{h}'\right)$ remains the same. The resource cost at t is γq^h while the resource benefit at t is the rent not payed, $q^h\left(1-\beta'\right)$ plus the extra amount the household can (and will, given the binding borrowing constraint) borrow, $\Delta b=\beta' m\gamma q^h$. The net resource gain **in the current period** is then

$$\Delta W_t=q^h\left(1-\beta'\right)+\beta' m\gamma q^h-\gamma q^h \tag{B.142}$$

From $t+1$ onwards, assuming nothing else changes, the household rolls over the debt and avoids paying rents. Therefore, the discounted value of all future gains starting in $t+1$ is

$$\begin{aligned} \Delta W_{\tau>0} &= \beta\sum_{\tau=0}^{\infty}\beta^{\tau}\left(q^h\left(1-\beta'\right)-\frac{1-\beta'}{\beta'}\beta' m\gamma q^h\right) \\ &= \frac{\beta}{1-\beta}q^h\left(1-\beta'\right)\left(1-m\gamma\right) \end{aligned} \tag{B.143}$$

Note that $\Delta W>0$ for $\gamma<1, m<1$. The net resouce change at t is positive if

$$\bar{\gamma}_t\equiv\frac{1-\beta'}{1-\beta' m}>\gamma$$

while the net (present value) of resource change is positive if

$$\begin{aligned} NPV\equiv\Delta W_t+\Delta W_{\tau>0} &> 0 \\ &\Leftrightarrow \\ \bar{\gamma}\equiv\frac{1-\beta'}{\left(1-\beta' m\right)\left(1-\beta\right)+m\beta\left(1-\beta'\right)} &> \gamma \end{aligned}$$

In other words, for $\gamma<\bar{\gamma}$, IHm households prefere to buy houses with a mortgage, while it is optimal for IHr households to rent, since for them, $\gamma=1$.

At the relative price $\frac{q^h}{p}$ implied by (B.141), a PH household is indifferent between buying an extra unit of housing stock to rent out to other other households, and not doing so (or lending it through the mortgage market). To see this, consider the following investment strategy: at t, buy one unit of housing at price q^h, and then rent it out. At $t+1$, sell that unit at price q^h. Given that the PH is not constrained in her borrowing capacity, she will not use this extra unit of housing as collateral to borrow. The net present value of this strategy in SS (recall again that the stochastic discount factor in SS is 1)

$$\begin{aligned} NPV &= q^h\left(1-\beta^{'}\right) - q^h + \beta^{'} q^h \\ &= 0 \end{aligned} \tag{B.144}$$

This is consistent with the condition $\lambda^{6'} = 0$ and $s^{-'} \geq 0$.

Is this investmet strategy optimal for a constrained IHm? Since the IHm household uses the housing stock as collateral for borrowing more today, the NPV of a buy-to-let strategy would be

$$\begin{aligned} NPV_{invest} &= q^h\left(1-\beta^{'}\right) - q^h + \beta^{'} mq + \beta \sum_{\tau=0}^{\infty} \beta^{\tau}\left(q^h\left(1-\beta^{'}\right) - mq^h\left(1-\beta^{'}\right)\right) \\ &= q^h\left(1-m\right)\frac{\beta-\beta^{'}}{1-\beta} < 0 \end{aligned} \tag{B.145}$$

since $\beta < \beta^{'}$. Therefore, if $p = q^h\left(1-\beta^{'}\right)$, IHm households will not engage in buy-to-let, and IHr can only rent from patient households. We have one possible SS:

$$\boxed{\left\{h>0, s^- = s^+ = 0\right\}; \left\{h^{'}>0, s^{-'}>0, s^{+'}=0\right\}; \left\{h^{''} = s^{-''} = 0, s^{+''}>0\right\}; p = q^h\left(1-\beta^{'}\right)}$$

– $\underline{\boldsymbol{\lambda^{6'} > 0}}$: From the KT conditions, this implies $s^{-'} = 0$, and from eqs. (B.138) and (B.140), we have that

$$p < q^h\left(1-\beta^{'}\right) \tag{B.146}$$

With this relative prices, IHr have even more incentives to rent. However, since $s^{-'} = 0$, they can only rent from IHm households. The individual feasibility condition (B.28) rules out sub-leting; therefore, if an IHm household rents to an IHr household, it has to do so from their stock of housing. However, since a buy-to-let strategy, which keeps the utility from housing constant, has a negative NPV_{invest} when $p = q^h\left(1-\beta^{'}\right)$ as shown in (B.145) above, it will be more negative for $p < q^h\left(1-\beta^{'}\right)$. Therefore, this can not be an equilibrium in SS.

- Case II: Assume $\boldsymbol{s^{+'} > 0}$

This implies $\lambda^{7'} = 0$ from the KT conditions. Linearity within the housing utility implies it cannot be optimal for a household to rent and buy at the same time.

Therefore it must be either that: (i) $h^{'} = s^{-'} > 0$, in which case the PH buys-to-let, or (ii) $h^{'} = s^{-'} = 0$. Lets consider these cases separately.

- $\underline{\boldsymbol{h}^{'} = \boldsymbol{s}^{-'} > \boldsymbol{0}}$: In this case we have $\lambda^{6'} = 0$. From equations (B.138) and (B.139) it follows that

$$p = q^h \left(1 - \beta^{'}\right) \tag{B.147}$$

  Someone has to rent to the PH. However, as shown above in eq (B.145), when $p = q^h \left(1 - \beta^{'}\right)$ it is not optimal for an IH household to buy-to-let. This cannot be an equilibrium.

- $\underline{\boldsymbol{h}^{'} = \boldsymbol{s}^{-'} = \boldsymbol{0}}$: This means that the PH does not own houses, and rents from other households. In this case we have $\lambda^{5'} \geq 0, \lambda^{6'} \geq 0$ and therefore

$$p \leq \gamma q^h \left(1 - \beta^{'}\right) \tag{B.148}$$

  Note, however, that the relative price $\frac{q^h}{p}$ implied by (B.148) is even bigger than the one implied by (B.147). Therefore, we can conclude that it wont be optimal for either IH household to rent to the PH households. This cannot then be an equilibrium.

<u>**Impatient Households' Problem**</u>

Equation (B.144) above showed that, when $p = q^h \left(1 - \beta^{'}\right)$, a buy-to-let strategy for the PH has NPV = 0. Note that the NVP of lemding 1 consumption unit through the mortgage market is also 0 in SS:

$$\begin{aligned} NPV^{mortgage} &= -1 + \beta^{'} R \\ &= -1 + \frac{\beta^{'}}{\beta^{'}} = 0 \end{aligned}$$

This implies that for $p > q^h \left(1 - \beta^{'}\right)$, a patient household in SS will prefer to invest all resources in a buy-to-let strategy, since in that case we would have

$$R^{buy-to-let} > R = \frac{1}{\beta^{'}}$$

and we are assuming no uncertainty. This would imply that the mortgage market would collaps unless house prices increase, since the PH would prefer to invest by a buy-to-let strategy rather than by mortgage lending. Moreover, since $R^{buy-to-let} \beta^{'} > 1$, consumption would grow unboundedly. This cannot be a SS equilibrium that satisfies a transversality condition.

We now proceed from the IHm households optimal conditions in order to check whether there is an alternative SS.

Without adjustment cost, the optimality conditions are

$$\begin{aligned}
\lambda_t q_t^h &= \frac{j_t}{\tilde{h}_t} + \beta E_t\left(\lambda_{t+1} q_{t+1}^h\right) + E_t\left(m\lambda_{BC,t} q_{t+1}^h \pi_{t+1}\right) + \lambda_t^5 \\
p_t \lambda_t &= \frac{j_t \gamma}{\tilde{h}_t} + \lambda_t^7 \\
p_t \lambda_t &= \frac{j_t}{\tilde{h}_t} + \lambda_t^5 - \lambda_t^6
\end{aligned}$$

which in SS imply

$$\begin{aligned}
\lambda q^h \left(1 - \beta - m(\beta' - \beta)\right) &= \frac{j}{\tilde{h}} + \lambda^5 \\
\lambda p &= \frac{j\gamma}{\tilde{h}} + \lambda^7 \\
\lambda p &= \frac{j}{\tilde{h}} + \lambda^5 - \lambda^6
\end{aligned}$$

- Case I: Assume $s^+ = 0$

  This implies $\lambda^7 \geq 0$, and due to the inada conditions on $u\left(\tilde{h}\right)$, it must be that $h - s^- > 0$, implying $\lambda^5 = 0$. Now we can consider two cases:

  - $s^- > 0$: then we have $\lambda^6 = 0$ which implies

    $$p = q^h \left(1 - \beta - m(\beta' - \beta)\right) \tag{B.149}$$

    It is easy to check that, for such relative rental rate, switching from one rental unit to γ units of housing stock has a positive net resource gain for the IHm for any value of $\gamma < 1$, confirming indeed that it is optimal for the IHm to have $s^+ = 0$. For the IHr, the net resource gain (or NPV) of switching is

    $$\begin{aligned}
    NPV &= q^h \left(1 - \beta - m(\beta' - \beta)\right) - q^h + \beta' m q^h \\
    &\quad + \frac{\beta}{1-\beta}\left(q^h \left(1 - \beta - m(\beta' - \beta)\right) - (1 - \beta')mq^h\right) \\
    &= 0
    \end{aligned}$$

    implying they are indifferent between renting and buying.
    As aargued above, however, the relaive prices in (B.149) imply that the PH would prefere oto engage in buy-to-let rather than lend through the mortgage market, and her consumption would grow unboundedly since the return of such strategy would satisfy $R^{buy-to-let}\beta' > 1$. This **cannot** be a SS equilibrium, since it would violate a transversality condition.

– $\underline{\boldsymbol{s^- = 0}}$: We then have $s^- = s^+ = 0$ and $\lambda^5 = 0$, $\lambda^6, \lambda^7 \geq 0$. From the FOCs in SS, we have that this is an optimal choice if

$$p = q^h \left(1 - \beta - m(\beta' - \beta)\right) - \frac{\lambda^6}{\lambda} \leq q^h \left(1 - \beta - m(\beta' - \beta)\right)$$

and

$$p = \gamma q^h \left(1 - \beta - m(\beta' - \beta)\right) + \frac{\lambda^7}{\lambda} \geq \gamma q^h \left(1 - \beta - m(\beta' - \beta)\right)$$

B.6 House prices and mortgage equity withdrawal

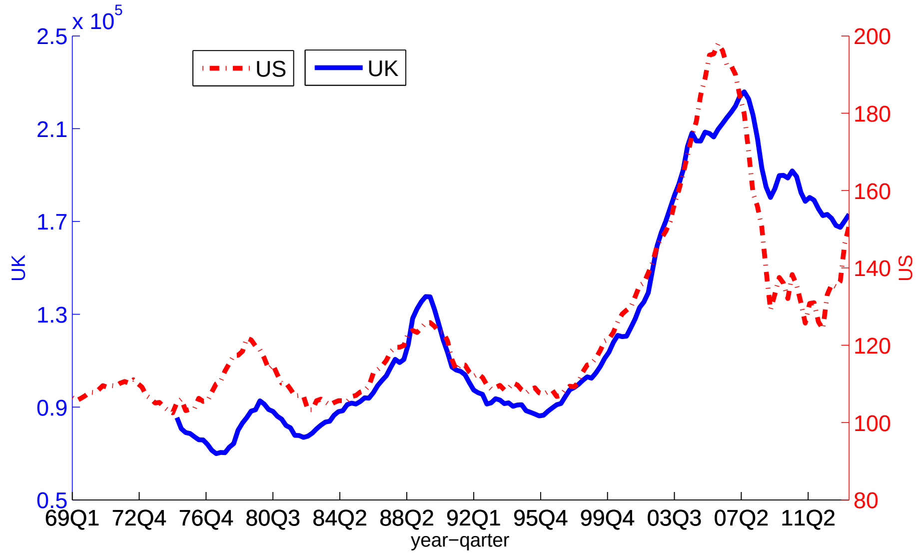


FIGURE B.12: Real House Price indexes. UK source: NationWide. US source: Case Shiller

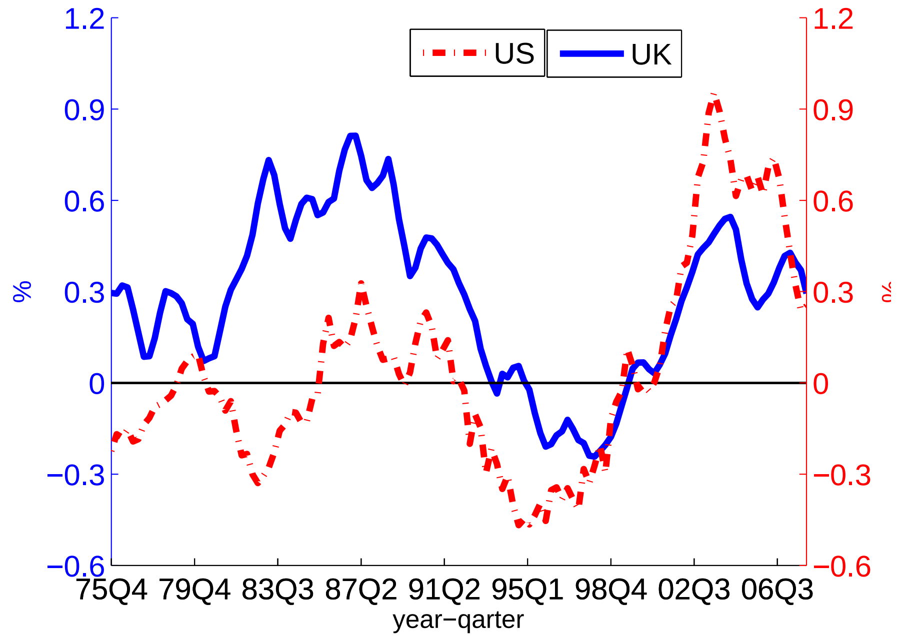


FIGURE B.13: Mortgage Equity Withdrawal as a % of Net Housing Wealth.

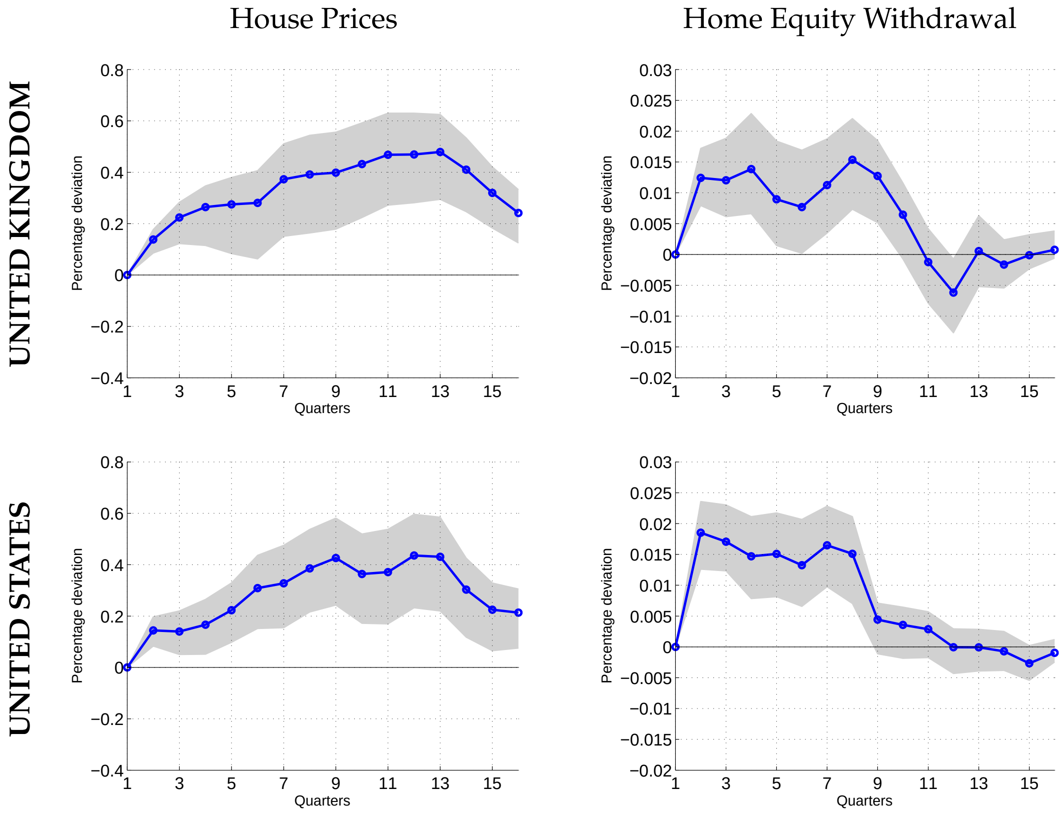


FIGURE B.14: IRF for aggregate real house prices (first column) and home equity withdrawal as a share of net housing wealth (second column). Grey areas are bootstrapped 90% confidence bands. Top row: U.K. (1975-2007). Bottom row: U.S. (1981-2007).

Appendix C

Appendix Chapter 3

C.1 Piecewise-linear Solution - Sketch of the Algorithm

The algorithm proposed in Guerrieri and Iacoviello, (2015b) builds on a non-linear approximation to the equilibrium decision functions of the model in section 3.4. Define and collect all the *endogenous* variables of the model in the vector X_t:

$$X_t \equiv \left(\left\{ C_t^i, D_t^i, V_t^i, h_t^i, b_t^i, L_t^i \right\}_{i=IH,PH}, q_t^h, q_t^d, w_t, R_t, S_t \right)$$

and the only shock, the monetary shock $\epsilon_{R,t}$ in the vector $u_t \equiv \epsilon_{R,t}$. Given the inequality constraint (3.5), one can define two "regimes" in the model economy: one where the constraint binds (the "reference" regime), and the other where it is slack. For each of these regimes, the economy is linearized around the *same* point, namely the 0-sectorial inflation steady state, in which the constraint binds. The linearized system in each case can be written as

$$A_1 E_t \left(X_{t+1} \right) + A_0 X_t + A_{-1} X_{t-1} + B u_t = 0 \tag{C.1}$$

if (3.5) binds, and

$$A_1^* E_t \left(X_{t+1} \right) + A_0^* X_t + A_{-1}^* X_{t-1} + B^* u_t + C^* = 0 \tag{C.2}$$

if it is slack, where $A_1, A_1^*, A_0, A_0^*, A_{-1}, A_{-1}^*, B, B^*, C^*$ are conformable matrices (vectors) and $E_t\left(\cdot\right)$ is the mathematical conditional expectation operator. When there is no shock, the economy is characterized by (C.1) and the solution can be written as[1]

$$X_t = P X_{t-1} + Q u_t \tag{C.3}$$

with P, Q matrices of *reduced form* parameters. Then, starting from an initial state X_0 where (3.5) binds, and **given** a shock u_1 in $t = 1$ (no other shocks are expected to hit the economy after $t = 1$), the algorithom uses a guess-and-verify strategy:

1. Guess the period T when the economy goes back to the reference regime , i.e. the constraint (3.5) binds again permanently after the shock of size u_1. Then for $t \geq T$, $X_t = P X_{t-1}$ from (C.3).

[1] Using any standard solution method for linear rational expectation systems.

2. At $T-1$, the constraint is assumed slack, and therefore the solution can be written by combining (C.2) and (C.3):

$$X_{T-1} = -\left(A_1^* P + A_0^*\right)^{-1} \left(X_{T-2} + C^*\right) \tag{C.4}$$

3. One can iterate backwards up to $t = 0$, starting from (C.4) and using in each step either (C.1) or (C.2) depending on the guess for the regime holding in each period. The result is a *time-varying* decision rule.

4. Using the solution obtained in steps 1-3, compute a path for X, and check whether the initial guess for the regimes holding in each period is verifyed. If it does, stop. Otherwise, go back to step 1.

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